**PROFESSOR:** Hi. Welcome back to recitation. We've been talking in lecture about various different applications of definite integrals. And one of them has been that we can use definite integrals to find areas of regions that previously we wouldn't have been able to. So, one simple example of that is that we can use a definite integral to find the area of a region bounded between two curves.

So I have an example of such a question right here. So the question is to compute the area of the region that's bounded between the curves y equals x cubed and y equals 3x minus 2. So one thing you'll notice is I haven't given you endpoints for this region. So I've just given you the bound and curves. So one thing you're going to have to do right at the beginning is to figure out what this looks like and what region you're going to be integrating over. What interval you're going to be integrating over.

So, this is a kind of tricky one. Why don't you pause the video, think about it for a few minutes, try and work it out yourself, and you can come back and we can try and work it out together.

All right, welcome back. So hopefully you've had some luck figuring out what this situation looks like and then computing the integral. So let's talk about it for a minute. So we have these two curves. And so somewhere-- I'm asking about the region bounded by them-- so what I'm telling you is somewhere these curves intercept and they surround some bounded region. And so I'm asking for what the area of that region is.

So in order to figure that out, we should figure out where these curves intercept. So to find the, in other words, we need to find the endpoints of the interval over which we're going to integrate. So in order to that, we have to solve for where we have the intersection between y equals x cubed and y equals 3x minus 2. So we have to solve the equation. So we need to solve the equation x cubed equals 3x minus 2. Or x cubed minus 3x plus 2 equals 0. So this a polynomial equation. It's not quadratic, right? It's a cubic equation. So that means it's hard to solve in general. Luckily in this case, it's-- there are some-- there's a root we can recognize fairly easily.

Which is, it's pretty-- you know, one thing you can always do is check small positive integers or use the rational root theorem, if you remember that from high school math. So in this case, if you check x equals 1, it's fairly easy to see that x equals 1 is a root of this equation. So in other words, we can factor the left-hand side. We can divide out a factor of x minus 1. OK, and so now we have to do long division or synthetic division, whatever kind of division you want, to divide through here.

So we get x squared minus-- so we've got a-- nope, I lied. Plus x minus 2 after you divide. How's that look? We can just double-check. You know, we get an x cubed minus x squared plus x squared minus 2x minus x plus 2. OK. So that adds up to the right thing. So we either have that 1 is a root or that x squared you know, rather for the places where these intersect, we have either x equals 1 or x squared plus x minus 2 equals 0. And now here you can, you know, again factor or use the quadratic equation or what have you.

And you can see so we have that this actually fully factors as x minus 1 squared times x plus 2 equals 0. Which means that we have intersections when either x minus 1 is 0 or when x plus 2 is 0. So intersections at x equals 1 and x equals minus 2. All right.

So you can take that information and you can, you know, put it together and you can make a nice picture like this. Or, I suppose, you could have made the nice picture before you had that information. And so we see that we've got this line, y equals 3x minus 2 here. And we have the curve y equals x cubed. And they have two intersection points. They intersect once down at x equals minus 2, y equals minus 8. And they intersect again up at the point (1, 1). And they're actually tangent there. So x cubed, the curve y equals x cubed stays above the line at this point. So one way you can read that off is here you had a double root at minus 1, if you like. But, OK, so then, the region in question is this region here. So it's a region bounded between those two curves that we're trying to compute the area of.

So now that we have the endpoints this isn't such a tricky problem at all. Right? so we've got the endpoints and we know-- now that we've drawn this picture-- we know which curve is on top and which curve is on bottom. Right? So the height, when we imagine cutting this region into lots of little rectangles or approximate rectangles, the height is going to be x cubed minus 3x minus 2. Minus the quantity 3x minus 2. Right? The x cubed is on top and 3x minus 2 is on the bottom.

So the area is equal to the integral. OK, and now we know where we have to integrate from and to. So we're integrating from x equals minus 2 to 1 to get this whole region of x cubed minus the quantity 3x minus 2 dx. OK? Because the y equals x cubed is the top curve. And y equals 3x minus 2 is the bottom curve. So this is the height of those rectangles, which is positive. OK, and so now, OK, well now this is pretty straightforward. We're integrating a polynomial at this point.

So this is the integral. Well, OK, so integrating x cubed, that gives me x to the fourth over 4. Integrating minus 3x gives me minus 3x squared over 2. And integrating plus 2 gives me plus 2x. Between x equals minus 2 and 1. So OK, so now I just do this difference. So this is equal to 1/4 minus 3/2 plus 2 minus-- OK, now I put in minus 2 I get 2 to the fourth-- sorry-- minus 2 to the fourth is 16, over 4, so that's minus 4. OK, minus 3 times 4 over 2 is 6. And then plus 2 times minus 2 is minus 4 again. OK, and now we've just got some arithmetic.

So this all becomes a plus 6. And I have to add everything together. So that's something like, well I've got a denominator of 4, so it's, yeah. All right, so there's a secret I should tell you. Which is that mathematicians are not actually very good at arithmetic, usually. So this is minus 6/4 plus 2 plus 6, so plus 8. So this is 8 minus 5/4, which is 6 and 3/4. OK, that's 6 plus 3/4.

So 6 and 3/4. This was just arithmetic. Here was the calculus part. And, in fact, you know, one feature of a problem like this is that you have a fair amount of work sometimes to see the picture of what you're working with. Then the integration here was pretty straightforward. Right? This was a fairly easy integral to compute. If you're, say, better at fraction arithmetic than I am, at least.

So, there you go. So, we had this region. So we found, actually the endpoints of the interval. We found which was the top curve and which was the bottom curve. And that led us right down this integral. And then that integral was fairly easy to compute from that point out. And then we ended up with this as our final area. So I'll end there.