

Average Height

Find the average height of a point on a unit semicircle.

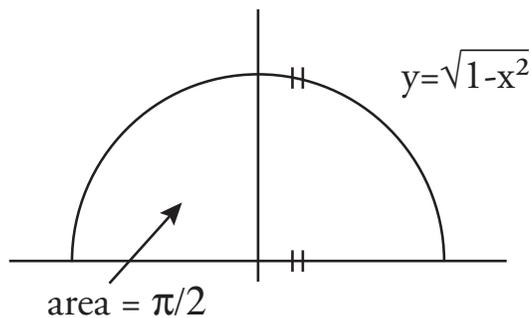


Figure 1: The unit semicircle and an interval dx .

Here $f(x) = \sqrt{1-x^2}$ for $-1 \leq x \leq 1$, so $a = -1$ and $b = 1$. The average value of $f(x)$ is:

$$\begin{aligned} \text{Avg}(f) &= \frac{1}{b-a} \int_a^b f(x) dx \\ &= \frac{1}{2} \int_{-1}^1 \sqrt{1-x^2} dx \\ &= \frac{1}{2} (\text{Area of a unit semicircle}) \\ &= \frac{1}{2} \left(\frac{\pi}{2} \right) \\ &= \frac{\pi}{4}. \end{aligned}$$

(We will eventually learn how to find the antiderivative of $\sqrt{1-x^2}$ in the unit on techniques of integration.)

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