

Volume of a Wine Glass: Horizontal Slices

Now we know all of the techniques of integration anyone knows. We'll celebrate by using our new techniques to answer an interesting question.

Find the volume of an exponential wine glass whose bowl is formed by rotating the portion of the graph of $y = e^x$ that joins $(0, 1)$ and $(1, e)$ about the y -axis.

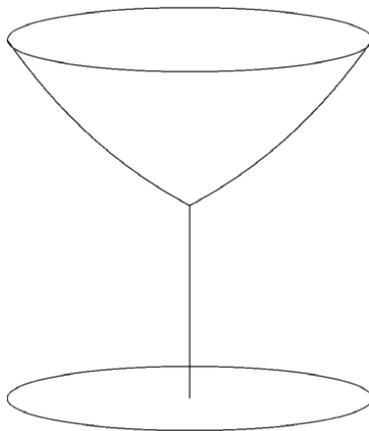


Figure 1: A wine glass formed by rotating the graph of $y = e^x$ about the y -axis.

There are two methods of solving this problem: horizontal and vertical slices. If we compute the volume using horizontal slices we'll be adding up the volumes of disks with height dy and radius $x = \ln y$. (See Figure 2.)

The volume will therefore be:

$$\begin{aligned} \int_1^e \pi x^2 dy &= \int_1^e \pi (\ln y)^2 dy \\ &= \pi F_2(y)|_1^e \quad (\text{see previous example}) \\ &= \pi [y(\ln y)^2 - 2(y \ln y - y)]_1^e \\ &= \pi [(e(\ln e)^2 - 2(e \ln e - e)) - (1(\ln 1)^2 - 2(1 \ln 1 - 1))] \\ &= \pi [(e(1)^2 - 2(e \cdot 1 - e)) - (1(0)^2 - 2(1 \cdot 0 - 1))] \\ &= \pi [(e - 2(0)) - (-2(-1))] \\ &= \pi(e - 2). \end{aligned}$$

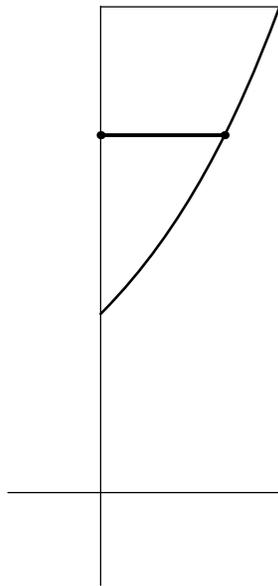


Figure 2: Rotating this horizontal slice about the y -axis forms a disk.

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