

Example: $y = mx$

This is a basic example that should help you get some perspective on this method.

$$\begin{aligned}y &= mx \\y' &= m \\ds &= \sqrt{1 + (y')^2} dx \\&= \sqrt{1 + m^2} dx\end{aligned}$$

The length of the curve $y = mx$ on the interval $0 \leq x \leq 10$ is:

$$\int_0^{10} \sqrt{1 + m^2} dx = 10\sqrt{1 + m^2}$$

We've drawn a picture to confirm this; see Figure 1. We see that the

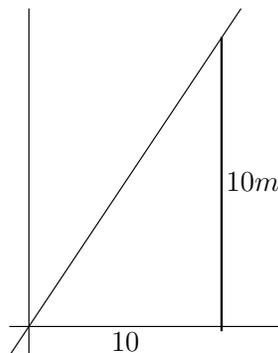


Figure 1: Arc Length of $y = mx$ over $0 \leq x \leq 10$.

arc whose length we're computing is the hypotenuse of a triangle, and the Pythagorean theorem tells us that its length is:

$$\sqrt{10^2 + (10m)^2} = 10\sqrt{1 + m^2}.$$

You may be disdainful of how obvious this is. But if we can figure out these formulas for linear functions, we can use calculus to subdivide other functions into infinitesimal linear parts and then solve the problem for those functions. This is the main point of these integrals.

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