

## Finding the Radius of Convergence

Use the ratio test to find the radius of convergence of the power series

$$\sum_{n=1}^{\infty} \frac{x^n}{n}.$$

### Solution

As Christine explained in recitation, to find the radius of convergence of a series  $\sum_{n=n_0}^{\infty} c_n x^n$  we apply the ratio test to find  $L = \lim_{n \rightarrow \infty} \left| \frac{c_{n+1} x^{n+1}}{c_n x^n} \right|$ . The value of  $x$  for which  $L = 1$  is the radius of convergence of the power series.

In this case,

$$\begin{aligned} \frac{c_{n+1} x^{n+1}}{c_n x^n} &= \frac{x^{n+1}/(n+1)}{x^n/n} \\ &= x \cdot \frac{n}{n+1}. \end{aligned}$$

Taking the limit of this as  $n$  goes to infinity, we get:

$$\begin{aligned} L &= \lim_{n \rightarrow \infty} \left| \frac{c_{n+1} x^{n+1}}{c_n x^n} \right| \\ &= \lim_{n \rightarrow \infty} \left| x \cdot \frac{n}{n+1} \right| \\ &= \lim_{n \rightarrow \infty} \left| x \cdot \left( 1 - \frac{1}{n+1} \right) \right| \\ L &= |x|. \end{aligned}$$

When  $|x| < 1$ ,  $L < 1$  and the ratio test tells us that the series will converge. For  $|x| > 1$ ,  $L > 1$  and the series diverges. The radius of convergence is 1.

This gives us an idea of how close the harmonic series  $\sum_{n=1}^{\infty} \frac{1}{n}$  is to being convergent.

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