So, last week we learned how to do triple integrals in rectangular and cylindrical coordinates.

And, now we have to learn about spherical coordinates, which you will see are a lot of fun.

So, what's the idea of spherical coordinates?

Well, you're going to represent a point in space using the distance to the origin and two angles.

So, in a way, you can think of these as a space analog of polar coordinates because you just use distance to the origin, and then you have to use angles to determine in which direction you're going.

So, somehow they are more polar than cylindrical coordinates.

So, how do we do that? So, let's say that you have a point in space at coordinates x, y, z.

Then, instead of using x, y, z, you will use, well, one thing you'll use is the distance from the origin.

OK, and that is denoted by the Greek letter which looks like a curly p, but actually it's the Greek R.

So -- That's the distance from the origin.

And so, that can take values anywhere between zero and infinity. Then, we have to use two other angles. And, so for that, let me actually draw the vertical half plane that contains our point starting from the z axis.

OK, so then we have two new angles.

Well, one of them is not really new.

One is new. That's phi is the angle downwards from the z axis. And the other one, theta, is the angle counterclockwise from the x axis. OK, so phi, let me do it better.

So, there's two ways to draw the letter phi, by the way. And, I recommend this one because it doesn't look like a rho.

So, that's easier. That's the angle that you have to go down from the positive z axis.

And, so that angle varies from zero when you're on the z axis, increase to pi over two when you are on the xy plane all the way to pi or 180ï¿½ when you are on the negative z axis. It doesn't go beyond that.

OK, so -- Phi is always between zero and pi.

And, finally, the last one, theta, is just going to be the same as before.

So, it's the angle after you project to the xy plane.

That's the angle counterclockwise from the x axis. OK, so that's a little bit overwhelming not just because of the new letters, but also because there is a lot of angles in there.

So, let me just try to, you know, suggest two things that might help you a little bit.

So, one is, these are called spherical coordinates because if you fix the value of rho, then you are moving on a sphere centered at the origin. OK, so let's look at what happens on a sphere centered at the origin, so, with equation rho equals a. Well, then phi measures how far south you are going, measures the distance from the North Pole. So, if you've learned about latitude and longitude in geography, well, phi and theta you can think of as latitude and longitude except with slightly different conventions. OK, so, phi is more or less the same thing as latitude in the sense that it measures how far north or south you are. The only difference is in geography, latitude is zero on the equator and becomes something north, something south, depending on how far you go from the equator.

Here, you measure a latitude starting from the North Pole which is zero, increasing all the way to the South Pole, which is at pi. And, theta or you can think of as longitude, which measures how far you are east or west. So, the Greenwich Meridian would be here, now, the one on the x axis.

That's the one you use as the origin for longitude, OK? Now, if you don't like geography, here's another way to think about it.

So -- Let's start again from cylindrical coordinates, which hopefully you're kind of comfortable with now.

OK, so you know about cylindrical coordinates where we have the z coordinates stay z, and the xy plane we do R and theta polar coordinates. And now, let's think about what happens when you look at just one of these vertical planes containing the z axis. So, you have the z axis, and then you have the direction away from the z axis, which I will call r, just because that's what r measures. Of course, r goes all around the z axis, but I'm just doing a slice through one of these vertical half planes, fixing the value of theta.

Then, r of course is a polar coordinate seen from the point of view of the xy plane. But here, it looks more like you have rectangular coordinates again.

So the idea of spherical coordinate is you're going to polar coordinates again in the rz plane.

OK, so if I have a point here, then rho will be the distance from the origin. And phi will be the angle, except it's measured from the positive z axis, not from the horizontal axis. But, the idea in here, see, let me put that between quotes because I'm not sure how correct that is, but in a way, you can think of this as polar coordinates in the rz plane. So, in particular, that's the key to understanding how to switch between spherical coordinates and cylindrical coordinates, and then all the way to x, y, z if you want, right, because this picture here tells us how to express z and r in terms of rho and phi.

So, let's see how that works. If I project here or here, so, this line is z. But, it's also rho times cosine phi. So, I get z equals rho cos phi.

And, if I look at r, it's the same thing, but on the other side. So, r will be rho sine phi.

OK, so you can use this to switch back and forth between spherical and cylindrical. And of course, if you remember what x and y were in terms of r and theta, you can also keep doing this to figure out, oops.

So, x is r cos theta. That becomes rho sine phi cos theta. Y is r sine theta.

So, that becomes rho sine phi sine theta.

And z is rho cos phi. But, basically you don't really need to remember these formulas as long as you remember how to express r in terms of rho sine phi, and x equals r cos theta. So, now, of course, we're going to use spherical coordinates in situations where we have a lot of symmetry, and in particular, where the z axis plays a special role.

Actually, that's the same with cylindrical coordinates.

Cylindrical and secure coordinates are set up so that the z axis plays a special role. So, that means whenever you have a geometric problem, and you are not told how to choose your coordinates, it's probably wiser to try to center things on the z axis. That's where these coordinates are the best adapted. And, in case you ever need to switch backwards, I just want to point out, so, rho is the square root of r squared plus z squared, which means it's the square root of x squared plus y squared plus z squared.

OK, so that's basically all the formulas about spherical coordinates. OK, any questions about that?

OK, let's see, who had seen spherical coordinates before just to see? OK, that's not very many.

So, I'm sure for, one of you saw it twice.

That's great. Sorry, oops, OK, so let's just look quickly at equations of some of the things. So, as I've said, if I set rho equals a, that will be just a sphere of radius a centered at the origin. More interesting things: let's say I give you phi equals pi over four.

What do you think that looks like?

Actually, let's take a quick poll on things.

OK, yeah, everyone seems to be saying it's a cone, and that's indeed the correct answer.

So, how do we see that? Well, remember, phi is the angle downward from the z axis.

So, let's say that I'm going to look first at what happens if I'm in the right half of a plane of a blackboard, so, in the yz plane. Then, phi is the angle downward from here. So, if I want to get pi over four, that's 45ï¿½. That means I'm going to go diagonally like this. Of course, if I'm in the left half of a plane of a blackboard, it's going to be the same.

I also take pi over four. And, I get the other half.

And, because the equation does not involve theta, it's all the same if I rotate my vertical plane around the z axis. So, I get the same picture in any of these vertical half planes, actually.

OK, now, so this is phi equals pi over four.

And, just in case, to point out to you what's going on, when phi equals pi over four, cosine and sine are equal to each other. They are both one over root two.

So, you can find, again, the equation of this thing in cylindrical coordinates, which I'll remind you was z equals r. OK, in general, phi equals some given number, or z equals some number times r. That will be a cone centered on the z axis. OK, a special case: what if I say phi equals pi over two?

Yeah, it's just going to be the xy plane.

OK, that's the flattest of all cones.

OK, so phi equals pi over two is going to be just the xy plane. And, in general, if phi is less than pi over two, then you are in the upper half space. If phi is more than pi over two, you'll be in the lower half space.

OK, so that's pretty much all we need to know at this point.

So, what's next? Well, remember we were trying to do triple integrals. So now we're going to triple integrals in spherical coordinates.

And, for that, we first need to understand what the volume element is. What will be dV?

OK, so dV will be something, d rho, d phi, d theta, or in any order that you want.

But, this one is usually the most convenient.

So, to find out what it is, well, we should look at how we are going to be slicing things now.

OK, so if you integrate d rho, d phi, d theta, it means that you are actually slicing your solid into little pieces that live, somehow, if you set an interval of rows, OK, sorry, maybe I should, so, if you first integrate over rho, it means that you will actually choose first the direction from the origin even by phi and theta. And, in that direction, you will try to figure out, how far does your region extend? And, of course, how far that goes might depend on phi and theta.

Then, you will vary phi. So, you have to know, for a given value of theta, how far down does your solid extend? And, finally, the value of theta will correspond to, in which directions around the z axis do we go?

So, we're going to see that in examples.

But before we can do that, we need to get the volume element. So, what I would like to suggest is that we need to figure out, what is the volume of a small piece of solid which corresponds to a certain change, delta rho, delta phi, and delta theta?

So, delta rho means that you have two concentric spheres, and you are looking at a very thin shell in between them.

And then, you would be looking at a piece of that spherical shell corresponding to small values of phi and theta.

So, because I am stretching the limits of my ability to draw on the board, here's a picture. I'm going to try to reproduce on the board, but so let's start by looking just at what happens on the sphere of radius a, and let's try to figure out the surface area elements on the sphere in terms of phi and theta.

And then, we'll add the rho direction.

OK, so -- So, let me say, let's start by understanding surface area on a sphere of radius a.

So, that means we'll be looking at a little piece of the sphere corresponding to angles delta phi and in that direction here delta theta. OK, so when you draw a map of the world on a globe, that's exactly what the grid lines form for you. So, what's the area of this guy?

Well, of course, all the sides are curvy.

They are all on the sphere. None of them are straight.

But still, if it's small enough and it looks like a rectangle, so let's just try to figure out, what are the sides of your rectangle? OK, so, let's see, well, I think I need to draw a bigger picture of this guy.

OK, so this guy, so that's a piece of what's called a parallel in geography. That's a circle that goes east-west. So now, this parallel as a circle of radius, well, the radius is less than a because if your vertical is to the North Pole, it will be actually much smaller. So, that's why when you say you're going around the world it depends on whether you do it at the equator or the North Pole. It's much easier at the North Pole. So, anyway, this is a piece of a circle of radius, well, the radius is what I would call r because that's the distance from the z axis.

OK, that's actually pretty hard to see now.

So if you can see it better on this one, then so this guy here, this length is r. And, r is just rho, well, what was a times sine phi.

Remember, we have this angle phi in here.

I should use some color. It's getting very cluttered.

So, we have this phi, and so r is going to be rho sine phi. That rho is a.

So, let me just put a sine phi. OK, and the corresponding angle is going to be measured by theta.

So, the length of this is going to be a sine phi delta theta.

That's for this side. Now, what about that side, the north-south side? Well, if you're moving north-south, it's not like east-west.

You always have to go all the way from the North Pole to the South Pole. So, that's actually a great circle meridian of length, well, I mean, well, the radius is the radius of the sphere.

Total length is 2pi a. So, this is a piece of a circle of radius a. And so, now, the length of this one is going to be a delta phi.

OK, so, just to recap, this is a sine phi delta theta.

And, this guy here is a delta phi.

So, you can't read it because it's -- And so, that tells us if I take a small piece of the sphere, then its surface area, delta s, is going to be approximately a sine phi delta theta times a delta phi, which I'm going to rewrite as a squared sine phi delta phi delta theta.

So, what that means is, say that I want to integrate something just on the surface of a sphere.

Well, I would use phi and theta as my coordinates.

And then, to know how big a piece of a sphere is, I would just take a squared sine phi d phi d theta.

OK, so that's the surface element in a sphere.

And now, what about going back into the third dimension, so, adding some depth to these things?

Well, I'm not going to try to draw a picture because you've seen that's slightly tricky. Well, let me try anyway just you can have fun with my completely unreadable diagrams.

So anyway, if you look at, now, something that's a bit like that piece of sphere, but with some thickness to it.

The thickness will be delta rho, and so the volume will be roughly the area of the thing on the sphere times the thickness.

So, I claim that we will get basically the volume element just by multiplying things by d rho.

So, let's see that. So now, if I have a sphere of radius rho, and another one that's slightly bigger of radius rho plus delta rho, and then I have a little box in here. Then, I know that the volume of this thing will be essentially, well, its thickness, the thickness is going to be delta rho times the area of its base, although it doesn't really matter, which is what we've called delta s.

OK, so we will get, sorry, a becomes rho now.

Square sine phi, delta rho, delta phi, delta theta, and so out of that we get the volume element and spherical coordinates, which is rho squared sine phi d rho, d phi, d theta. And, that's a formula that you should remember. OK, so whenever we integrate a function, and we decide to switch to spherical coordinates, then dx dy dz or r dr d theta dz will become rho squared sine phi d rho d phi d theta.

OK, any questions on that? No?

OK, so let's -- Let's see how that works.

So, as an example, remember at the end of the last lecture, I tried to set up an example where we were looking at a sphere sliced by a slanted plane.

And now, we're going to try to find the volume of that spherical cap again, but using spherical coordinates instead. So, I'm going to just be smarter than last time. So, last time, we had set up these things with a slanted plane that was cutting things diagonally. And, if I just want to find the volume of this cap, then maybe it makes more sense to rotate things so that my plane is actually horizontal, and things are going to be centered on the z axis. So, in case you see that it's the same, then that's great. If not, then it doesn't really matter. You can just think of this as a new example. So, I'm going to try to find the volume of a portion of the unit sphere -- -- that lies above the horizontal plane, z equals one over root two.

OK, one over root two was the distance from the origin to our slanted plane. So, after you rotate, that say you get this value. Anyway, it's not very important.

You can just treat that as a good example if you want.

OK, so we can compute this in actually pretty much any coordinate system. And also, of course, we can set up not only the volume, but we can try to find the moment of inertia about the central axis, or all sorts of things. But, we are just doing the volume for simplicity. So, actually, this would go pretty well in cylindrical coordinates.

But let's do it in spherical coordinates because that's the topic of today. A good exercise: do it in cylindrical and see if you get the same thing.

So, how do we do that? Well, we have to figure out how to set up our triple integral in spherical coordinates.

So, remember we'll be integrating one dV.

So, dV will become rho squared sign phi d rho d phi d theta.

And, now as we start, we're already facing some serious problem. We want to set up the bounds for rho for a given, phi and theta.

So, that means we choose latitude/longitude.

We choose which direction we want to aim for, you know, which point of the sphere we want to aim at.

And, we are going to shoot a ray from the origin towards this point, and we want to know what portion of the ray is in our solid. So -- We are going to choose a value of phi and theta. And, we are going to try to figure out what part of our ray is inside this side.

So, what should be clear is at which point we leave the solid, right? What's the value of rho here?

It's just one. The sphere is rho equals one.

That's pretty good. The question is, where do we enter the region? So, we enter the region when we go through this plane. And, the plane is z equals one over root two. So, what does that tell us about rho? Well, it tells us, so remember, z is rho cosine phi.

So, the plane is z equals one over root two.

That means rho cosine phi is one over root two.

That means rho equals one over root two cosine phi or, as some of you know it, one over root two times second phi. OK, so if we want to set up the bounds, then we'll start with one over root two second phi all the way to one. Now, what's next?

Well, so we've done, I think that's basically the hardest part of the job. Next, we have to figure out, what's the range for phi? So, the range for phi, well, we have to figure out how far to the north and to the south our region goes. Well, the lower bound for phi is pretty easy, right, because we go all the way to the North Pole direction. So, phi starts at zero.

The question is, where does it stop?

To find out where it stops, we have to figure out, what is the value of phi when we hit the edge of the region?

OK, so maybe you see it. Maybe you don't.

One way to do it geometrically is to just, it's always great to draw a slice of your region. So, if you slice these things by a vertical plane, or actually even better, a vertical half plane, something to delete one half of the picture. So, I'm going to draw these r and z directions as before. So, my sphere is here.

My plane is here at one over root two.

And, my solid is here. So now, the question is what is the value of phi when I'm going to stop hitting the region?

And, if you try to figure out first what is this direction here, that's also one over root two.

And so, this is actually 45ï¿½, also known as pi over four.

The other way to think about it is at this point, well, rho is equal to one because you are on the sphere.

But, you are also on the plane. So, rho cos phi is one over root two. So, if you plug rho equals one into here, you get cos phi equals one over root two which gives you phi equals pi over four.

That's the other way to do it. You can do it either by calculation or by looking at the picture.

OK, so either way, we've decided that phi goes from zero to pi over four. So, this is pi over four.

Finally, what about theta? Well, because we go all around the z axis we are going to go just zero to 2pi.

OK, any questions about these bounds?

OK, so note how the equation of this horizontal plane in spherical coordinates has become a little bit weird.

But, if you remember how we do things, say that you have a line in polar coordinates, and that line does not pass through the origin, then you also end up with something like that. You get something like r equals a second theta or a cos second theta for horizontal or vertical lines. And so, it's not surprising you should get this. That's in line with the idea that we are just doing again, polar coordinates in the rz directions. So of course, in general, things can be very messy.

But, generally speaking, the kinds of regions that we will be setting up things for are no more complicated or no less complicated than what we would do in the plane in polar coordinates. OK, so there's, you know, a small list of things that you should know how to set up. But, you won't have some really, really strange thing. Yes?

D rho? Oh, you mean the bounds for rho?

Yes. So, in the inner integral, we are going to fix values of phi and theta.

So, that means we fix in advance the direction in which we are going to shoot a ray from the origin.

So now, as we shoot this ray, we are going to hit our region somewhere. And, we are going to exit, again, somewhere else. OK, so first of all we have to figure out where we enter, where we leave.

Well, we enter when the ray hits the flat face, when we hit the plane. And, we would leave when we hit the sphere. So, the lower bound will be given by the plane. The upper bound will be given by the sphere. So now, you have to get spherical coordinate equations for both the plane and the sphere. For the sphere, that's easy.

That's rho equals one. For the plane, you start with z equals one over root two.

And, you switch it into spherical coordinates.

And then, you solve for rho. And, that's how you get these bounds. Is that OK?

All right, so that's the setup part.

And, of course, the evaluation part goes as usual.

And, since I'm running short of time, I'm not going to actually do the evaluation. I'm going to let you figure out how it goes. Let me just say in case you want to check your answers, so, at the end you get 2pi over three minus 5pi over six root two.

Yes, it looks quite complicated. That's basically because you get one over, well, you get a second square when you integrate C. When you integrate rho squared, you will get rho cubed over three.

But that rho cubed will give you a second cube for the lower bound. And, when you integrate sine phi second cubed phi, you do a substitution.

You see that integrates to one over second squared with a factor in front. So, in the second square, when you plug in, no, that's not quite all of it.

Yeah, well, the second square is one thing, and also the other bound you get sine phi which integrates to cosine phi. So, anyways, you get lots of things. OK, enough about it.

So, next, I have to tell you about applications.

And, of course, well, there's the same applications that we've seen that last time, finding volumes, finding masses, finding average values of functions.

In particular, now, we could say to find the average distance of a point in this solid to the origin.

Well, spherical coordinates become appealing because the function you are averaging is just rho while in other coordinate systems it's a more complicated function. So, if you are asked to find the average distance from the origin, spherical coordinates can be interesting. Also, well, there's moments of inertia, preferably the one about the z axis because if you have to integrate something that involves x or y, then your integrand will contain that awful rho sine phi sine theta or rho sine phi cosine theta, and then it won't be much fun to evaluate.

So, that anyway, there's the usual ones.

And then there's a new one. So, in physics, you've probably seen things about gravitational attraction.

If not, well, it's what causes apples to fall and other things like that as well.

So, anyway, physics tells you that if you have two masses, then they attract each other with a force that's directed towards each other. And in intensity, it's proportional to the two masses, and inversely proportional to the square of the distance between them.

So, if you have a given solid with a certain mass distribution, and you want to know how it attracts something else that you will put nearby, then you actually have to, the first approximation will be to say, well, let's just put a point mass at its center of mass. But, if you're solid is actually not homogenous, or has a weird shape, then that's not actually the exact answer.

So, in general, you would have to just take every single piece of your object and figure out how it attracts you, and then compute the sum of these. So, for example, if you want to understand why anything that you drop in this room will fall down, you have to understand that Boston is actually attracting it towards Boston.

And, Somerville's attracting it towards Somerville, and lots of things like that. And, China, which is much further on the other side is going to attract towards China.

But, there's a lot of stuff on the other side of the Earth.

And so, overall, it's supposed to end up just going down. OK, so now, how to find this out, well, you have to just integrate over the entire Earth.

OK, so let's try to see how that goes.

So, the setup that's going to be easiest for us to do computations is going to be that we are going to be the test mass that's going to be falling. And, we are going to put ourselves at the origin. And, the solid that's going to attract us is going to be wherever we want in space.

You'll see, putting yourself at the origin is going to be better. Well, you have to put something at the origin. And, the one that will stay a point mass, I mean, in my case not really a point, but anyway, let's say that I'm a point.

And then, I have a solid attracting me.

Well, so then if I take a small piece of it with the mass delta M, then that portion of the solid exerts a force on me, which is going to be directed towards it, and we'll have intensity.

So, the gravitational force -- -- exerted by the mass delta M at the point of x, y, z in space on a mass at the origin. Well, we know how to express that. Physics tells us that the magnitude of this force is going to be, well, G is just a constant. It's the gravitational constant, and its value depends on which unit system you use.

Usually it's pretty small, times the mass delta M, times the test mass little m, divided by the square of the distance. And, the distance from U to that thing is conveniently called rho since we've been introducing spherical coordinates.

So, that's the size, that's the magnitude of the force. We also need to know the direction of the force. And, the direction is going to be towards that point. So, the direction of the force is going to be that of x, y, z.

But if I want a unit vector, then I should scale this down to length one. So, let me divide this by rho to get a unit vector. So, that means that the force I'm getting from this guy is actually going to be G delta M m over rho cubed times x, y, z.

I'm just multiplying the magnitude by the unit vector in the correct direction. OK, so now if I have not just that little p is delta M, but an entire solid, then I have to sum all these guys together.

And, I will get the vector that gives me the total force exerted, OK? So, of course, there's actually three different calculations in one because you have to sum the x components to get the x components of a total force. Same with the y, and same with the z. So, let me first write down the actual formula. So, if you integrate over the entire solid, oh, and I have to remind you, well, what's the mass, delta M of a small piece of volume delta V? Well, it's the density times the volume. So, the mass is going to be, sorry, density is delta. There is a lot of Greek letters there, times the volume element. So, you will get that the force is the triple integral over your solid of G m x, y, z over rho cubed, delta dV.

Now, two observations about that.

So, the first one, well, of course, these are just constants. So, they can go out.

The second observation, so here, we are integrating a vector quantity. So, what does that mean?

I just mean the x component of a force is given by integrating G m x over rho cubed delta dV. The y components, same thing with y. The z components, same thing with z. OK, there's no, like, you know, just integrate component by component to get each component of the force.

So, now we could very well to this in rectangular coordinates if we want. But the annoying thing is this rho cubed. Rho cubed is going to be x squared plus y squared plus z squared to the three halves.

That's not going to be a very pleasant thing to integrate.

So, it's much better to set up these integrals in spherical coordinates. And, if we're going to do it in spherical coordinates, then probably we don't want to bother too much with x and y components because those would be unpleasant. It would give us rho sine phi cos theta or sine theta. So, the actual way we will set up things, set things up, is to place the solid so that the z axis is an axis of symmetry.

And, of course, that only works if the solid has some axis of symmetry. Like, if you're trying to find the gravitational attraction of the Pyramid of Giza, then you won't be able to set up so that it has rotation symmetry. Well, that's a tough fact of life, and you have to actually do it in x, y, z coordinates. But, if at all possible, then you're going to place things.

Well, I guess even then, you could center it on the z axis. But anyway, so you're going to mostly place things so that your solid is actually centered on the z-axis. And, what you gain by that is that by symmetry, the gravitational force will be directed along the z axis. So, you will just have to figure out the z component. So, then the force will be actually, you know in advance that it will be given by zero, zero, and some z component. And then, you just need to compute that component. And, that component will be just G times m times triple integral of z over rho cubed delta dV. OK, so that's the first simplification we can try to do. The second thing is, well, we have to choose our favorite coordinate system to do this. But, I claim that actually spherical coordinates are the best -- -- because let's see what happens. So, G times mass times triple integral, well, a z in spherical coordinates becomes rho cosine phi over rho cubed.

Density, well, we can't do anything about density. And then, dV becomes rho squared sine phi d rho d phi d theta.

Well, so, what happens with that?

Well, you see that you have a rho, a rho squared, and a rho cubed that cancel each other.

So, in fact, it simplifies quite a bit if you do it in spherical coordinates.

OK, so the z component of the force, sorry, I'm putting a z here to remind you it's the z component.

That is not a partial derivative, OK?

Don't get things mixed up, just the z component of the force becomes Gm triple integral of delta cos phi sine phi d rho d phi d theta. And, so this thing is not dV, of course. dV is much bigger, but we've somehow canceled out most of dV with stuff that was in the integrand. And see, that's actually suddenly much less scary. OK, so just to give you an example of what you can prove it this way, you can prove Newton's theorem, which says the following thing.

It says the gravitational attraction -- -- of a spherical planet, I should say with uniform density, or actually it's enough for the density to depend just on distance to the center.

But we just simplify the statement is equal to that of a point mass -- -- with the same total mass at its center.

OK, so what that means is that, so the way we would set it up is u would be sitting here and your planet would be over here.

Or, if you're at the surface of it, then of course you just put it tangent to the xy plane here. And, you would compute that quantity. Computation is a little bit annoying if a sphere is sitting up there because, of course, you have to find bounds, and that's not going to be very pleasant.

The case that we actually know how to do fairly well is if you are just at the surface of the planet.

But then, what the theorem says is that the force that you're going to feel is exactly the same as if you removed all of the planet and you just put an equivalent point mass here. So, if the earth collapsed to a black hole at the center of the earth with the same mass, well, you wouldn't notice the difference immediately, or, rather, you would, but at least not in terms of your weight. OK, that's the end for today.