FIRST PRACTICE MIDTERM
MATH 18.022, MIT, AUTUMN 10

You have 50 minutes. This test is closed book, closed notes, no calculators.

Name:_____

Signature:_____

Recitation Time:_____

There are 5 problems, and the total number of points is 100. Show all your work. *Please make your work as clear and easy to follow as possible.*

Problem	Points	Score
1	20	
2	20	
3	20	
4	20	
5	20	
Total	100	

1. (20pts) (i) Let $\vec{u}$ and $\vec{v}$ be two vectors. Show that the vectors $\vec{a} = \|\vec{u}\|\vec{v} + \|\vec{v}\|\vec{u}$ and $\vec{b} = \|\vec{u}\|\vec{v} - \|\vec{v}\|\vec{u}$ are orthogonal.

(ii) Show that the vector $\vec{a} = \|\vec{u}\|\vec{v} + \|\vec{v}\|\vec{u}$ bisects the angle between $\vec{u}$ and $\vec{v}$.

2. (20pts) (i) Find the equation of the plane through the three points $P_0 = (1, 1, 2)$, $P_1 = (-1, 2, -2)$ and $P_2 = (2, -1, 1)$.

(ii) Find the distance between this plane and the point $Q = (1, 1, 1)$.

3. (20pts) (i) What is the angle between the diagonal of a cube and one of the edges it meets?

(ii) Find the angle between the diagonal of a cube and the diagonal of one of its faces.

4. (20pts) Let D be the region inside the paraboloid $a^2z = x^2 + y^2$ and outside the sphere of radius a centred at the origin.
(i) Describe the region D in cylindrical coordinates.

(ii) Describe the region D in spherical coordinates.

5. (20pts) Determine whether or not the following limits exist, and if they do exist, then find the limit. Explain your answer.

(i) $\lim_{(x,y)\to(0,0)} \dfrac{x^4 - y^4}{x^2 + y^2}$

(ii) $\lim_{(x,y)\to(0,0)} \dfrac{x^2}{x^2 + y^2}$

18.022 Calculus of Several Variables
Fall 2010