PSET 2 - DUE FEBRUARY 15

1. Let $T_\theta : \mathbb{R}^2 \to \mathbb{R}^2$ be the linear transformation that takes a vector x to its rotation (counterclockwise) by θ degrees about the origin.

a. Find the matrix representation for T_θ using the standard basis for $\mathbb{R}^2$ ($\{(1,0),(0,1)\}$). (4 pts)

b. This matrix is obviously invertible. Find its inverse and verify by matrix multiplication. (2 pts)

2.Let $T : \mathbb{R}^3 \to \mathbb{R}^2$ and $S : \mathbb{R}^3 \to \mathbb{R}^3$ correspond to the transformations

$$T(x,y,z) = (x,y);\ S(x,y,z) = (-x,-y,-z).$$

Notice that TS is a well defined linear transformation. Find a matrix representation for S, T, and TS using the basis $\{(1,0,0),(1,1,0),(1,1,1)\}$ for $\mathbb{R}^3$ and the basis $\{(1,0),(1,-1)\}$ for $\mathbb{R}^2$. (6 pts)

3. 2.20:9 (4 pts)

4. For an $n \times n$ matrix A, we define $\lambda \in \mathbb{R}$ to be an *eigenvalue* of A if $Ax = \lambda x$ for some $x \neq 0 \in \mathbb{R}^n$.

a. Prove that λ is an eigenvalue for A if and only if it solves $det(A - \lambda I_n) = 0$. (Here I_n represents the $n \times n$ identity matrix.) (6 pts)

b. For

$$A = \begin{pmatrix} 4 & 1 & -2 \\ 16 & -2 & -8 \\ 4 & -2 & -2 \end{pmatrix}$$

determine the eigenvalues. (2 pts)

c. Use the results from part b to explain why A is not invertible. (2 pts)

5. Let X, Y be $n \times n$ matrices such that $X^3 = Y^3$ and $X^2Y = Y^2X$. What are necessary and sufficient conditions on X and Y such that $X^2 + Y^2$ is invertible? (4 pts)

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