PSET 6 - DUE MARCH 17

1. 8.22: 14 (5 points)

2. 8.24: 12 (5 points)

3. Let $f(x,y) = \int_0^{xy} g(u)du$ where $g : \mathbb{R} \to \mathbb{R}$ is a strictly positive continuous function.
   - Find $\nabla f : \mathbb{R}^2 \to \mathbb{R}^2$ in terms of g.
   - Consider a level set $\{(x,y) \in \mathbb{R}^2 | f(x,y) = c\}$. Prove that for a fixed $c \neq 0$ there are exactly two level curves in the set. Moreover, prove they are precisely the graph of the function $h(x) = b/x$ for exactly one $b \in \mathbb{R}$. (Do not try to determine b in terms of g! Just prove it exists and is unique!)
   - Parameterize one curve on a level set and prove that ∇f is orthogonal to the level set at each point on the curve.

(6 points)

4. Let $f : \mathbb{R}^2 \to \mathbb{R}$ by

(1)
$$f(x,y) = \begin{cases} xy\frac{x^2-y^2}{x^2+y^2} & (x,y) \neq (0,0) \\ 0 & (x,y) = (0,0) \end{cases}$$

   - Prove $\frac{\partial f}{\partial x}(0,y) = -y$ for any y and $\frac{\partial f}{\partial y}(x,0) = x$ for any x.
   - Prove $\frac{\partial^2 f}{\partial y \partial x} \neq \frac{\partial^2 f}{\partial x \partial y}$.

(6 points)

4. C20:5 (4 points)

5. C20:6 (4 points)

18.024 Multivariable Calculus with Theory
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