EXAM 1 - MARCH 4, 2011

Name:

(1) (10 points) Consider the transformation $T : \mathbb{R}^3 \to \mathbb{R}^3$ such that $T(1,0,0) = (2,1,4), T(0,1,0) = (4,3,6), T(0,0,1) = (0,-1,2)$.
  (a) Determine the null space of T.
  (b) If A is the plane formed by $span(\{(2,5,-3),(-1,-1,1)\})$, write $T(A)$ in parametric form.

(2) (10 points) Let

$$F(t) = \begin{cases} (\sin t, -\cos t) & t \in [0, \pi] \\ (\sin t, \cos t + 2) & t \in (\pi, 2\pi] \end{cases}$$

(a) Find $F'(\pi)$, if it is well defined.
(b) Find $F''(\pi)$, if it is well defined.
(c) Determine $\kappa(t)$ everywhere it is defined.

(3) (10 points) Let $f(x, y, z) = x^2 + y^2 + z^2$. Prove f is differentiable at $(1, 1, 1)$ with linear transformation $T(x, y, z) = 2x + 2y + 2z$.

(4) (15 points) Consider the set $\mathcal{L}(\mathbb{R}^3, \mathbb{R}^2)$ of all linear maps L from $\mathbb{R}^3$ to $\mathbb{R}^2$ and define addition of $L, K \in \mathcal{L}(\mathbb{R}^3, \mathbb{R}^2)$ the following way:

$$(L + K)(v) = L(v) + K(v) \qquad (v \in \mathbb{R}^3)$$

Define multiplication by a constant c as:

$$(cL)(v) = c(L(v)) \qquad (v \in \mathbb{R}^3)$$

(a) Are the linear maps $L(x, y, z) = (x, 0)$, $K(x, y, z) = (y, 0)$, $N(x, y, z) = (x, y)$ linearly independent? Prove it either way.
(b) Find a basis for $\mathcal{L}(\mathbb{R}^3, \mathbb{R}^2)$.
(c) What is the dimension of $\mathcal{L}(\mathbb{R}^3, \mathbb{R}^2)$?

(5) (15 points) Consider the function $f\colon \mathbb{R}^2 \to \mathbb{R}$ that satisfies the following conditions:

(a) For all fixed $x_0 \in \mathbb{R}$ the function $f_{x_0} = f(x_0, y)\colon \mathbb{R} \to \mathbb{R}$ is continuous and;

(b) For all fixed $y_0 \in \mathbb{R}$ the function $f^{y_0} = f(x, y_0)\colon \mathbb{R} \to \mathbb{R}$ is continuous and;

(c) For all fixed $x_0 \in \mathbb{R}$ the function f_{x_0} is monotonically increasing in y, i.e. if $y > y'$ then, $f(x_0, y) > f(x_0, y')$.

Prove f is continuous.