

18.02 Exam 1 Solutions

Problem 1.

a) $P = (1, 0, 0)$, $Q = (0, 2, 0)$ and $R = (0, 0, 3)$. Therefore $\overrightarrow{QP} = \hat{i} - 2\hat{j}$ and $\overrightarrow{QR} = -2\hat{j} + 3\hat{k}$.

$$\text{b) } \cos \theta = \frac{\overrightarrow{QP} \cdot \overrightarrow{QR}}{|\overrightarrow{QP}| |\overrightarrow{QR}|} = \frac{\langle 1, -2, 0 \rangle \cdot \langle 0, -2, 3 \rangle}{\sqrt{1^2 + 2^2} \sqrt{2^2 + 3^2}} = \frac{4}{\sqrt{65}}$$

Problem 2.

a) $\overrightarrow{PQ} = \langle -1, 2, 0 \rangle$, $\overrightarrow{PR} = \langle -1, 0, 3 \rangle$.

$$\overrightarrow{PQ} \times \overrightarrow{PR} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ -1 & 2 & 0 \\ -1 & 0 & 3 \end{vmatrix} = 6\hat{i} + 3\hat{j} + 2\hat{k}.$$

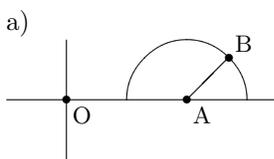
Then $\text{area}(\Delta) = \frac{1}{2} |\overrightarrow{PQ} \times \overrightarrow{PR}| = \frac{1}{2} \sqrt{6^2 + 3^2 + 2^2} = \frac{1}{2} \sqrt{49} = \frac{7}{2}$.

b) A normal to the plane is given by $\vec{N} = \overrightarrow{PQ} \times \overrightarrow{PR} = \langle 6, 3, 2 \rangle$. Hence the equation has the form $6x + 3y + 2z = d$. Since P is on the plane $d = 6 \cdot 1 + 3 \cdot 1 + 2 \cdot 1 = 11$. In conclusion the equation of the plane is

$$6x + 3y + 2z = 11.$$

c) The line is parallel to $\langle 2 - 1, 2 - 2, 0 - 3 \rangle = \langle 1, 0, -3 \rangle$. Since $\vec{N} \cdot \langle 1, 0, -3 \rangle = 6 - 6 = 0$, the line is parallel to the plane.

Problem 3.



$\overrightarrow{OA} = \langle 10t, 0 \rangle$ and $\overrightarrow{AB} = \langle \cos t, \sin t \rangle$, hence

$$\overrightarrow{OB} = \overrightarrow{OA} + \overrightarrow{AB} = \langle 10t + \cos t, \sin t \rangle.$$

The rear bumper is reached at time $t = \pi$ and the position of B is $(10\pi - 1, 0)$.

b) $\vec{V} = \langle 10 - \sin t, \cos t \rangle$, thus

$$|\vec{V}|^2 = (10 - \sin t)^2 + \cos^2 t = 100 - 20 \sin t + \sin^2 t + \cos^2 t = 101 - 20 \sin t.$$

The speed is then given by $\sqrt{101 - 20 \sin t}$. The speed is smallest when $\sin t$ is largest i.e. $\sin t = 1$. It occurs when $t = \pi/2$. At this time, the position of the bug is $(5\pi, 1)$. The speed is largest when $\sin t$ is smallest; that happens at the times $t = 0$ or π for which the position is then $(0, 0)$ and $(10\pi - 1, 0)$.

Problem 4.

a) $|M| = -12$.

b) $a = -5$, $b = 7$.

$$\text{c) } \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \frac{1}{12} \begin{bmatrix} 1 & 1 & 4 \\ -5 & 7 & -8 \\ 7 & -5 & 4 \end{bmatrix} \begin{bmatrix} 0 \\ t \\ 3 \end{bmatrix} = \begin{bmatrix} t/12 + 1 \\ 7t/12 - 2 \\ -5t/12 + 1 \end{bmatrix}$$

$$\text{d) } \frac{d\vec{r}}{dt} = \left\langle \frac{1}{12}, \frac{7}{12}, -\frac{5}{12} \right\rangle.$$

Problem 5.

a) $\vec{N} \cdot \vec{r}(t) = 6$, where $\vec{N} = \langle 4, -3, -2 \rangle$.

b) We differentiate $\vec{N} \cdot \vec{r}(t) = 6$:

$$0 = \frac{d}{dt} (\vec{N} \cdot \vec{r}(t)) = \frac{d}{dt} \vec{N} \cdot \vec{r}(t) + \vec{N} \cdot \frac{d}{dt} \vec{r}(t) = \vec{0} \cdot \vec{r}(t) + \vec{N} \cdot \frac{d}{dt} \vec{r}(t) \quad \text{and hence } \vec{N} \perp \frac{d}{dt} \vec{r}(t).$$

MIT OpenCourseWare
<http://ocw.mit.edu>

18.02SC Multivariable Calculus
Fall 2010

For information about citing these materials or our Terms of Use, visit: <http://ocw.mit.edu/terms>.