Tangent approximation

1. Find the equation of the tangent plane to the graph of $z = xy^2$ at the point (1,1,1).

Answer: $\dfrac{\partial z}{\partial x} = y^2$ and $\dfrac{\partial z}{\partial y} = 2xy \Rightarrow \dfrac{\partial z}{\partial x}(1,1) = 1$ and $\dfrac{\partial z}{\partial y}(1,1) = 1$.

The tangent plane at (1,1,1) is

$$(z-1) = \left.\frac{\partial z}{\partial x}\right|_0 (x-1) + \left.\frac{\partial z}{\partial y}\right|_0 (y-1) = (x-1) + 2(y-1).$$

2. Give the linearization of $f(x,y) = \mathrm{e}^x + x + y$ at (0,0).

Answer: The tangent approximation formula at the point (x_0, y_0, z_0) is

$$f(x,y) - f(x_0,y_0) \approx f_x(x_0,y_0)(x-x_0) + f_y(x_0,y_0)(y-y_0).$$

(We usually abbreviate this as $\Delta z \approx f_x|_0 \Delta x + f_y|_0 \Delta y$.)

Linearization is just the following form of the tangent approximation formula

$$f(x,y) \approx f(x_0,y_0) + f_x(x_0,y_0)(x-x_0) + f_y(x_0,y_0)(y-y_0).$$

In our case,

$$f_x(x,y) = \mathrm{e}^x + 1 \text{ and } f_y(x,y) = 1 \Rightarrow f(0,0) = 1,\ f_x(0,0) = 2,\ f_y(0,0) = 1$$

Thus, for $(x,y) \approx (0,0)$ we have

$$f(x,y) \approx 1 + 2x + y.$$