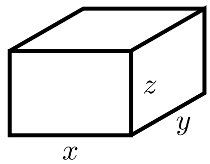
Lagrange multipliers

1. In an open-top wooden drawer, the two sides and back cost \$2/sq. ft., the bottom \$1/sq. ft. and the front \$4/sq. ft. Using Lagrange multipliers find the dimensions of the drawer with the largest capacity that can be made for \$72.

<u>**Answer:**</u> The box shown has dimensions x, y, and z.



The area of each side $= yz$; the area of the front (and back) $= xz$; the area of the bottom $= xy$. Thus, the cost of the wood is

$$C(x, y, z) = 2(2yz + xz) + xy + 4xz = 4yz + 6xz + xy = 72.$$

This is our constraint. We are trying to maximize the volume

$$V = xyz.$$

The Lagrange multiplier equations are then

$$\begin{aligned} & \boldsymbol{\nabla} V = \lambda \boldsymbol{\nabla} C, \text{ and } C = 72 \\ \Leftrightarrow \quad & \langle yz,\, xz,\, xy\rangle = \lambda\langle 6z + y,\, 4z + x,\, 4y + 6x\rangle, \quad 4yz + 6xz + xy = 72. \end{aligned}$$

We solve for the critical points by isolating $1/\lambda$.

$$\frac{1}{\lambda} = \frac{6}{y} + \frac{1}{z} = \frac{4}{x} + \frac{1}{z} = \frac{4}{x} + \frac{6}{y}$$

Comparing the third and fourth terms gives $\dfrac{1}{z} = \dfrac{6}{y} \;\Rightarrow\; y = 6z$.
Likewise the second and fourth terms give $x = 4z$.
Substituting this in the constraint gives $72z^2 = 72 \;\Rightarrow\; z = 1$. Thus,

$$z = 1,\; x = 4,\; y = 6.$$