NAME:

18.075 In–class Exam # 1
Wednesday, September 29, 2004

Justify your answers. Cross out what is not meant to be part of your final answer. Total number of points: 45.

I. (5 pts) Show that for any complex numbers z_1 and z_2,

$$||z_1| - |z_2|| \leq |z_1 + z_2|.$$

II. (5 pts) Find all possible values of

$$(-\sqrt{3}+i)^{1/5}.$$

III.

1. (3 pts) Can the function $u(x, y) = x^2 - y^2 - x - y$ be the REAL part of an analytic function $f(z) = u(x, y) + iv(x, y)$? **Hint:** You may use the Laplace equation, if you wish.

2. (5 pts) Determine all functions $v(x, y)$ such that $f(z) = u(x, y) + iv(x, y)$ is analytic.

3. (3 pts) Find explicitly as a function of z the $f(z)$ such that

$$f(z) = u(x, y) + iv(x, y).$$

IV. (6 pts) Compute the line integral

$$\int_C \frac{(z^3 + z^2 + z + 1)}{z^4}\, dz$$

where C is the LOWER half–circle centered at 0 joining $\frac{-1-i}{\sqrt{2}}$ and $\frac{1-i}{\sqrt{2}}$ in the positive (counterclockwise) sense.

V. Let

$$f(z) = \frac{1}{(2-z)(z+3)}.$$

1. (2 pts) Write $f(z)$ as a sum of fractions, i.e.,

$$f(z) \;=\; \frac{A}{z-2} + \frac{B}{z+3};$$

2. (3 pts) Explain whether it is possible to expand f(z) in Laurent (or Taylor) power series of:

   (i) z, that converges in $0 \leq |z| < 3$?

(ii) z, that converges in $3 < |z|$?

(iii) $z + 1$, that converges in $1 < |z + 1| < 4$?

3. (4 pts) Write the Laurent series expansion of $f(z)$ for $\underline{5 < |z-2| < \infty}$ as a power series of $(z-2)$.

VI. (6 pts) Let

$$f(z) = \frac{1}{(z^2+z)(z+2)^3}.$$

Compute the integral of $f(z)$ on the circles of <u>center 1</u> and radii $1/2$, $3/2$, and 100, respectively.

VII. (3 pts) Determine where in the complex plane the following functions are analytic ($\bar{z}$ is the complex conjugate of z):

(i) $\frac{e^z}{\sin z}$

(ii) $z(\bar{z} + i)$

(iii) $e^{\frac{1}{z-1}}$

VIII. (3 pts-BONUS) Determine the constant A so that the following function is analytic everywhere.

$$f(z) = \begin{cases} A\,\frac{\cosh z - 1}{z^2} & \text{if } z \neq 0 \\ 1 & \text{if } z = 0. \end{cases}$$