18.075 In-class Practice Test for Exam 2

October 29, 2004

Justify your answers. Cross out what is not meant to be part of your solution.

I. (10pts) By use of contour integration, evaluate the real integral

$$\int_0^\infty \frac{\cos x}{1+x^2}\,dx.$$

II. Consider the integral

$$\int_{-\infty}^{\infty} \frac{\cos x}{(4x^2-\pi^2)(x^2+4)}\,dx.$$

1. (2pts) By replacing x by the complex variable z, locate and characterize all singularities of the integrand (viewed as a function of z).

2. (3pts) Define indented contours in order to evaluate the given integral. In particular, state which parts of the contours finally contribute zero and why.

3. (10pts) Evaluate the requisite integral by use of the residue theorem.

III. Consider the real integral

$$\int_0^\pi \frac{\sin^2\theta}{2+\cos^2\theta}\,d\theta.$$

1. (5pts) Give suitable change(s) of variable to get a closed-contour integral in the complex plane.

2. (10pts) Evaluate the requisite integral by use of the residue theorem.

IV. Find the region of convergence of the following series by using the ratio or Cauchy (root) test.

1. (4pts)

$$\sum_{n=0}^{\infty} \frac{(-2)^n\,(x+1)^n}{n!}.$$

2. (4pts)

$$\sum_{n=0}^{\infty} a_n x^n,$$

where $a_n = n(n+1)$ for n: even and $a_n = n^2$ for n: odd.

V. (12pts) Describe how you would proceed to calculate the integral

$$\int_0^\infty \frac{x}{1+x^{2n+1}}\,dx,$$

where n is a positive integer. (You don't need to carry out the integration explicitly, but if you do you get extra credit.)