PROFESSOR STRANG: Okay, so this is I could say delta function day. Break from linear algebra mostly. So we're looking on another type of right-hand side. Before in the differential equation and in the difference equation. So the right-hand sides up to now, the one we looked at was a uniform constant load second derivative equal one. Now a point load. Well in a way, we're now solving a whole bunch of problems because the point load can be in different places. So instead of solving one problem with one on the right-hand side, we're solving with a delta function.

Now a delta function is, you probably have seen and heard the words and seen the symbol, but maybe not done much with a delta function. It takes a little practice but it's really worth it. It's a great model of maybe what can't quite happen physically, to have a load acting exactly at a point and nowhere else. So the delta function is, I drew it's picture, the delta function is zero, this is delta of x is zero except at that one point, the origin, x=0, and then all along back to zero again. So nothing's happening, no load except at that one point. And let me just, so there's no hesitation in when I change from x to x-a, what does that do to a graph? If I have a function of x and I instead shift the function to f(x-a), I shift x to x-a, well in this case, and in all cases, it will just shift the graph. So if I drew a picture of delta, of x-a, the load now would happen when this is zero, because it's delta at zero is the impulse, and now this is zero at x=a. In other words, the load moved to the point a. So there is the shifting load, but the load could fall anywhere between zero and one. So delta of x, the load actually falls at zero. Well we don't quite want that load at the boundary. So let's think of the point a, the load point as somewhere between zero and one.

Can I just take a little time to recall the main fact about delta functions? When I say recall, it could very well be new to you. So that's what the delta function-- that's my best graph of the delta function. But of course I'm, in using the word function, I'm kind of breaking the rules because no function-- I mean the function is, functions can be zero there, can be zero there, but they're not supposed to be infinite at a single point in between, but this one is. Let me go back to delta of x to match these figures. Of course, they would also just shift along by a. Maybe no harm in that. Sorry, I'll stay there and now I want to integrate. And that's when a delta function comes into its own. Its value of infinity is a little bit uncertain. What does that mean? But when we integrate it, what's the key fact about delta function? That the integral of a delta function from, let's say, let's integrate the whole thing, we can safely start way at the far left and go away to the far right because it's zero all the time there except at one point, and you know. So what's the area under that spike? It is one. That's right.

So that's the fact, the sort of central fact about a delta function. That the area is one. Oh well, let me, while I'm really writing down the central fact, let me write it more specifically, more generally. Suppose I integrate, and this is delta functions now really showing up, if I integrate a delta function against some, times some nice function. Now have you ever thought about that? What would be the answer if I integrate the delta function against some nice function? So I'm still getting zero from this term all the way along until I hit the spike and then after it goes back to zero again. So, whatever, it's gotta be at the spike, at x=0, because I put the spike here at zero, the impulse. So what do you think's the answer for that one? Yeah, It's the function. So, yes, tell me again and I'll write it down. g, it's a value of this function g. We don't care what it is to the left and to the right at zero because it's really at zero that this thing turns on and its value at that point is just-- gives us the amplitude of the impulse, which is g(0). And of course if g is the constant function one, I'm back to that formula. But this is maybe the thing to watch for. Actually there's a lot built into that little thing. We'll come back to that.

So that's delta functions integrated and now here are some pictures. These are the good pictures. So here's one integral of the delta function. It's a step function. And the step of course will occur at the point a if the integral of the delta function at a point a will be the step function. Where the action happens. The jump happens, I could call it a jump function. At that point a. Because, just for the reason we said. That if we integrate, the integral is zero. And then as soon as our integral passes this point, so this is integral of the, this is-- I integrated. I integrate to get to this picture. I start with that delta function and I integrate and it suddenly jumps to one as soon as the integral goes past the spike, the impulse.

So a step function. Very handy function, step function. Sometimes called a Heaviside function named after the guy who-- the electrical engineer I think who first sort of work out the rules for using these. Let's integrate one more time because we have second order equations, second derivatives, so we better integrate twice to see what sort of answer we get. Now integrate the step function. So again, the integral is zero all the way to the left, so I'm still getting zero, but now beyond this point I'm integrating one. And the integral of one is x. So now that I would call a ramp function. That's a nice short word for this valuable function. A ramp function is the function that's zero and then x. So tell me about that ramp function. Just think about it. What happens to its derivative at the point a? As I run along and I hit this key point, what happens to the derivative of the ramp? What does the derivative do? Focus on that ramp now. What does the derivative do at that point? It jumps. The derivative jumps. The slope is the derivative, the slope jumps from zero and here the slope is one. And of course that's what that's telling us. Here's the picture of the derivative.

What does the second derivative do? Well, since I integrated twice I guess going back two steps I'll find out what the second derivative is. So the first derivative takes a jump. The second derivative is the derivative of that jump, so it's got the impulse. So the second derivative, it's a straight line here, second derivative a straight line. This is straight line here, second derivative of a straight line is a straight line. But at that point the first derivative jumps, the second derivative has that delta function. In other words, that's that stuff. If I keep integrating -- and I don't need higher integrals in today's lecture -- another integral would be what? If I integrate this function, then it's running along the zero. What's the integral of this? Doesn't quite turn that steeply. What's that curve there? If I've integrated the ramp. Here is the integral. First, the next step up, the integral of the ramp would be? It'll be x squared, yeah, it'll be a parabola. x squared over two, the integral of that. And now what do I get when I integrate this one? I get something very important. Not important today, but important in a few weeks. And very useful in computing. These have turned out to be just the right thing. So again, I'm integrating that. Everybody can tell me, what is that? What's that curve now? It's the next integral of course. The area under that will be x cubed over six.

So now that is a function. Yeah, it's worth maybe just for practice. What's the deal with that function? That's pretty smooth function. Because it certainly passes-- right, it meets at that point. The first derivative meets at that point. The second derivative meets at that point. The third derivative does what? Of this line. The third derivative, take three steps back down the line and you see that the third derivative jumps. Right? The third derivative of that is the third derivative, would be, shall I-- for C, for cubic spline or something, the third derivative will be zero there. And the third derivative of that is exactly like back to that, back to that, back to one is one. So the third derivative. so the cubic spline's so smooth your eye doesn't see that. They're very useful for drawing many, many purposes. CAD programs would use such things constantly because they're convenient, they have nice pieces that you can fit together and they fit together very smoothly. But they really are two separate functions. So that's up to cubic spline. But our focus is--

These would solve, what equations would those solve? Well, that takes how many derivatives to get to a delta? So what would be the equation? What would be the right-hand side? Let me take the fourth derivative. I'll just ask the question that way. What would be the fourth derivative of that cubic spline? A delta, right? Four steps back. So what is, physically, what are we seeing here? Do you recognize what kind-- If I ask now people in mechanics, When will we meet a fourth order equation? Fourth derivative equals a load. Anybody know the physical situation where fourth derivative? Beams, yeah. It's the equation for a beam. A beam has-- The bending of a beam. So it's a beam. This eraser isn't too very much like a beam, but anyway I put the chalk on it, well nothing happened. Sit on it, whatever. It'll bend and that bending will be given by a beam equation. So later we'll meet the beam equation. So most equations of physics, mechanics, biology, everything are second order, Newton's Laws often the reason. But we get up to fourth order sometimes. And very seldom get higher. Hopefully. Beams or plates, that table would be a plate and it would have a fourth order equation.

Let's start solving this problem. What's the solution, what's the general solution to that equation? Minus the second derivative, so notice the minus that I like, and the load has now moved to the point a. So the solution u(x), let's write down all solutions. Tell me one solution, first. One particular solution. What is one function for which minus the second derivative would be the delta? That's what we've got over there. So just bring that blackboard over here. Change its sign because that minus, and what are you going to tell me? Minus a ramp. Minus a ramp. And the ramp, of course, will ramp up at the point a so that it's the second derivative of that, the second derivative of R will be delta. The minus is correct and the point is correct. Now does that solve our problem? No. The ramp is going upwards. It's not zero. What am I forgetting? What do I not yet have? There's more to this solution. Just as there was for a uniform load. What was the more? Constant and-- and I want two homogeneous solutions, null solutions, two solutions with second derivative equal zero. One of them is C and the other one is Dx. That's the whole solution. So what I want to-- I mean we need that C+Dx. We've got two boundary conditions to satisfy, just as before. So I need two constants, that'll do it perfectly and I'll get an exact answer. And so this is a ramp.

Oh yeah. Before I go further, how would I think about this? This is a ramp that turns which way? Down. Right? With that minus sign, that ramp turns down at the point x=a. Right? It's derivative goes from zero to minus one. The slope of this guy drops by one because of the minus sign. Sorry the slope of the ramp function, of minus the ramp. It goes at zero, drops by one. And what this is going to do is take that ramp and adjust it to go through the fixed ends. Oh, let's just do it. Let's just do it. What are C and D? What are C and D? My point a-- let me draw a graph, that's always the best thing. Always draw a graph of these solutions. So let me put in the point a. So I'm drawing now a picture of the solution from zero to one. I'll graph it. What do I have here? Shall we just plug in the boundary conditions and find C and D? That's the direct way. What is C? C I'm going to plug in. Hopefully I might find it from just the first boundary condition. If I'm starting from zero, well this guy certainly starts at zero, right? The ramp hasn't done anything until it gets to a. And this guy is certainly zero. So what is C? Gone, right. Now what is D? Well alright, what's D? Let's see. Let me draw the-- So there's a Dx, and D won't be zero. I want that thing to be zero at point one. So I want to determine D. Let me determine D. So what is minus the ramp at x=1? I'm plugging in x=1. Is that right? I'm going straight forward here. Plugging in x=1 into this boundary condition, ready for this guy. What's the ramp? So it's minus and the ramp is, well if the ramp is shifted over then that's shifted over. So at x=1, what's the ramp? How high has that ramp gone? 1-a. Right? The ramp is x-a. At the point x=1 it will be 1-a. So I think I get one, -(1-a) out of that. Minus the ramp plus D times what? One, I'm plugging in x=1. And that's supposed to equal? Zero, good.

So I'm doing this sort of the systematic way of writing down the general solution. Discovering that D, what do I discover D is? Put it on the other side. D is 1-a. And of course, don't forget that it's multiplying the x. Let me just draw the picture. Here's how I think about it. The solution is, away from x=a, what does the solution look like? To the left of x=a what's my graph going to be? It's going to be? A straight line, right? To the left of here there is no load. The equation is second derivative equals zero. The solution to that is a straight line. In other words, until I get to a, this thing hasn't started. It's only this straight line. The solution does something like that. It's a straight line. And I guess, actually, that's what it is. Because the C isn't here and that's all we've got left. So that's that straight line.

What is it for the second half? Tell me what the solution looks like in the second half. In between a and one. It's going downhill. Why? Because it gotta get back to zero. And how's it going downhill? It has to be linear. In this region, has to be linear. Why? How do I know it's linear here? Because one way is to say the equation in that region is second derivative equal zero. Second derivative equal zero, straight line. This is my solution. It's (1-a)x here and it's whatever it is to get back to zero. What will it take to get back to zero? Let's see. Well we could plug in, we've got one expression here. Or I could just look at that. I could say, okay what's the equation for the straight line that's at this point, what is the, yeah, it's (1-a)x. I want it to be linear. I want it to get to zero. Let's see. If I want that, it would be great to have 1-x times something. I have to figure out what. Because with the 1-x at x=1, that'll drop off. That's linear. What number, what's the key here? That slope, I want to match them up there. And that's the point x=a. This is supposed to match that at x=a. Do you have an idea for what I should take? What do I put right there? a. Look at the symmetry in those two sides. (1-a)x going up. (1-x)a going down. At x=a it hits that point, right.

So we've solved it. We could think about this different ways. I could have got that 1-x, let's see, I could have got it from the formula. In a way I like to get it from the picture, I see it, sort of, I see the point. What happened at that point? What are the jump conditions? This is another way to ask, to see how the delta function works. What are they jump conditions? I want to know, when I ask about jump conditions, I want to know what are the conditions on u(x), the displacement? What are the conditions on the slope, u'(x)? That'll be the strain when we're speaking about elasticity. Just for u(x), what's the statement about u(x) from the left and from the right at that critical point, the point of the load. From the left and from the right u(x) is? The same. u(x) matches up. u(x) from the left is that height. u(x) from the right is that.

I want to write down those jump conditions. Because that's another way to see this. u(x), u(a) from the left should equal u-- do you want me to say u is continuous? I'll just say it in words. u(x) is continuous, that just means it doesn't jump, at x=a. So that's, you could say that's a non-jump condition. The function itself doesn't jump. Why not? Because we're talking about some elastic bar on which we put a point load. The thing isn't going to break. The displacement is going to be continuous. But what's the condition on u'(x), the derivative, the slope? So that's the function and now tell me what's the deal on the slope? What's the comparison between the-- I have a slope of whatever it is going along here and I have a slope of-- a new slope. So u'(x), the slope jumps, right? And how much does it jump? Minus one. It drops by one. The slope, because of my minus. So this tells me that-- Yeah, let me write that down. u'(x) drops by one.

This is another way to say what the equation is asking. The equation is looking for two pieces of straight lines that meet at a but their slope drops by one. By the way, what were the slopes? It's good to graph the slopes, too. Let me graph the slopes. The slope u', the derivative du/dx. What's the slope here? Slope is 1-a at this point, right? The derivative is 1-a along here. So slope is 1-a. And now at x=a the slope changes to this one. And what's the slope of that second part? Minus a. Look. It did it right. Minus a is the slope along here. Do you see 1-a? It dropped by one. The one disappeared to leave a slope of minus a.

I guess if I just imagine a bar. I'm fixing it at both ends. There's a bar. I'm just thinking for people who like to see a physical picture of what's happening, that's what this is, we'll do it properly very, very soon. I've got a bar. It's a very light bar. Its weight is not a problem here. But it's got a load at the point. So I'll measure x going downwards. And at the point x=a I'm hanging a heavy load. A load. How do I draw a load? Maybe I'll make a big weight or something. What's going to happen to this dumb bar when I do that? Just tell me physically. What's going to happen? What's going to happen above the load? It's going to stretch, right, tension. The load is going to pull the bar down, it's going to stretch this part. And because nothing special is happening, it's going to stretch it linearly. And then what's going to happen below the load? Compression. So the slope will go negative. And nothing special happened so the slope will be negative but it'll be constant. The slope will drop from this to this. The displacement, that point will go down a little bit. That little bit it goes down is actually the height of this, because that's the displacement. It'll go down a little bit. It'll stretch above, it'll compress below, and we see that in that picture of the displacement. The displacement's all down. Right? Displacement-- You know, nature is still going to-- All the bar is going to move down. That's why this function doesn't, this function, the displacement function is positive. It goes all down. But the slope function is positive here, so tension is positive slope, stretch. And compression is negative. Well all that to solve this equation.

Maybe while we're on a roll, let's solve the free-fixed guy. So this is our-- might as well be systematic. This is the fixed-fixed problem. Let me below it solve the free-fixed problem. So it'll be minus u'', that's the second derivative, equals delta at x-a. Same setup. But now the top end is, so it's free at the top. What does that mean? Slope is zero at the top but it's still fixed at the bottom. So this will be now free-fixed. Let me go straight to the picture. Let me go straight to the picture of u(x). So there is x=0, there's x=1, here's the load at a. What's up? And while you're thinking about that, let me draw a picture to match this picture. A bar fixed at the bottom but not at the top. And it's got its load here hanging down. But let's do it math first, and then check with the picture.

What have we got, two or three ways now to try to get the answer? The systematic way would be to write down this solution and plug in the two boundary conditions. That'd be a straightforward way. Yeah, we could even start by that. So u(x) is the particular solution, the ramp plus any Cx+D. And just plug in x=0 that'll be easy. If I plug in x=0 in the free condition, what does that tell me? At x=0, this corner, this ramp hasn't started so the slope is zero. The slope of the constant is zero. What do I learn from this boundary condition? u'(0)=0. That C is zero. Before I learned that D was zero, but now from that condition I'm going to learn C is zero.

Do the picture for me. Do the picture for me. What's the graph of-- this is a graph of u(x). Remember now it starts from zero slope because it's free at the top. What does the graph look like in the first part? It's a straight line, has to be a straight line because there's no force. And what kind of a line? It's going to be horizontal because it starts off horizontal. The slope has to be zero at zero and nothing changes until a. So it comes along there. Right? Now I've started out with the right, left, the correct boundary condition at zero, which was no slope. And now what's it going to do the other half? From a to one. It's going to be again, it'll be a straight line, right? Because there's no force there. And what happens at-- all the action of course is at this point a, and what action is it? Tell me what sort of a line. How do I finish the picture? What do I do? I start here, right? Because the bar's not falling apart. u is continuous. I don't get a gap suddenly. And now what do I do from there? Only thing I can possibly do, because I have to end up here and it has to be a straight line, that's it. That's what the picture will have to look like. What does that correspond to in the picture for the bar? Well what happens with this bar? Above the weight, what happens to this top part of the bar in that picture? And what happens to the lower part of the bar? So this was at the point x=a, this is x=0, this is x=1. What happens above the bar, above the weight? It just-- A rigid motion, just goes down. Because what happens below the weight? The same compression or compression still happening. This is still squeezed. Shall I try to draw it? So this is after the weight. This got squeezed but this part did not get squeezed. And that's what we're seeing here. A fixed displacement. So this means, that picture means that all the pieces of the bar here got moved down by the same amount, whatever this, we don't know that number yet. And then below it they got compressed. Well we're almost there but we don't yet have that solution.

Come back to this picture. u(x) is continuous, got it. And what's the real condition that's going to determine where we are, what those heights are, the numbers in there. It's gotta look like that, but we get more than that, we gotta know what are the actual, what is that height. What is this? What's the slope? Here the slope is zero. Here the slope is what? What's the slope in the second part? That's the key. And you know what it has to be because what happens to the slope? If I have the second derivative as a delta function with that minus sign, the slope drops by one. And the slope here is zero, so the slope here is minus one. And now it has to get through there, so what is the function? What's the function that has a slope of minus one and comes down to zero? It's gotta have a minus x in it and what's the constant to make it come out right? What do I write now here for u(x)? 1-x. That has a slope of minus one, the derivative is minus one, at x=1 it comes to zero, that's it. And what do I write, what's u(x) up here? And therefore, right there? What's the displacement there, of all this bit that moves down, how much does it move down? 1-a. Why 1-a? That's the right answer. 1-a. Why's that? Because it had to match up at x=a. At x=a, this and that match up. At x=a, that slope, that function and that function match up.

So the slope picture is zero and-- Oh, I'm sorry, can't draw it because I'm at the bottom of the board. The slope picture, maybe I can draw it here, the slope picture is zero along here and then it drops by one to 1-a. So that's a picture of u'. Zero and minus one. This is the thing to look at. That's hard work, when you're seeing delta functions the first time. But of course the functions did not get complicated. We kept a clean example. And which we matched up with a figure and we've got the answer and we've got a couple of ways to do it. One is this standard, systematic, plug-in boundary condition way. The other way is this. u(x) does something here, then the slope has to drop by one. And that's the key to everything with a boundary condition. So in a way, we have a piece to the left and a piece to the right. Two constants here, two constants here, and somewhere there are four conditions that settle those four constants. You know, we could have a straight line here, a straight line here, that's two and two. But what are the four conditions that settle those four constants? Well we have a boundary condition here, that's one. Boundary condition here is two. We need two more conditions to settle the two pairs of constants, and there they are. Two conditions at the jump, at the discontinuity.

Now I've got to do the discrete case. Are you up for the discrete case? The case where we're doing-- We have a difference equation, so we're doing Ku equal a column of the identity. Column of I. Let me take a specific column. Say [0, 1, 0, 0, 0]. Let's suppose we have five-- I'm going to draw a picture now. We have five, because I made it five by five. One, two, three, four, five, here is zero and here is six. So h is 1/(5+1), 1/6, that's the delta x. And my equation says-- So what does my equation say? Remember what K is. u is then u_1, u_2, u_3, u_4, and u_5, the unknowns. K is our old friend with twos and minus ones and minus ones. I'm going to find the solution. And this'll be the solution that has a load at this point. This is like my point a, right? Here in the continuous case, a could run anywhere between zero and one. In the discrete case, I've got five possible load points and I've picked the second one. Five columns of the identity matrix, five places to put that one, I put it there.

Now can I draw the picture here? Which should we do first? Should we do free-fixed? Because that came out even easier than fixed-fixed. Notice the solution here had two parts. This is the way I would write that answer. Because you could draw a picture, but if you want to write the formula, what would I do? I would break it into two pieces. 1-a up to the point a because that's what it was running along here. And then down here it was 1-x, x >=a. That's important to mention. You have to have some guidance on how to write the answer. And when the answer has two parts, this is a good way to write it, in two parts. It's a little too-- you're compressing it too much to write, to use that ramp function. Better to split it apart into before a and after a. What's going to happen over here?

Oh yeah, can we take a shot at this problem? And let me mention again, in the review that'll be in here this afternoon and every Wednesday afternoon I'll just be ready for questions. Please bring questions. They can be questions on the homework. Even better if they're questions on other problems, questions on the lecture. Questions are essential to make that help session helpful.

What do you think's cooking here? At a typical-- Somewhere in the middle here, I'm going to draw the u's. Shall I just draw them? And now what's my condition? I gotta put the boundary conditions on. Oh, I have put the boundary conditions on it. By putting that two there, I'm up to here. Okay, let's do that one. When I chose K and put a two in there I was picking the fixed-fixed boundary condition. So can I just say it's going to be beautiful. The solution over there is going to look like this. The solution over here is going to be up, up, up. It's going to be a straight line but only points in a line and it'll be straight line down. That value, that value, that value. Those will be u_1, u_2, u_3, u_4, and u_5. And once more, this is going to drop by one again. Actually I didn't have to redraw the picture. It falls right on. In case x is 2/6 so that it fits that picture, I'm claiming we have another extremely lucky case. If we can use the word lucky for math.

I'm claiming that the way the-- You remember for the uniform load with a one, when we had second derivative equal one, the solution was a perfect parabola and the discrete solution, the difference equation was right on the parabola for this fixed-fixed case. It's going to happen again. It won't always happen. Those are the only two important right-hand sides I know. They're the two most important right-hand sides and those are the two lucky ones. If we have a constant that lies right on a parabola, if we have a delta function, it lies right on a ramp. And there it is. So that's what the solution looks like. Now, I have to figure out what these numbers are, I guess. Yes, what are those numbers? Oh, well. Actually, if it falls right on, I know the numbers. So a is 2/6. So let me keep 2/6. So a is 2/6. That's that value. So let me say what I think u is. So this was a picture of u. That's u_1, 2, 3, 4, and 5 and now I think it lies right on that. So it's going to be (1-2/6)x going up and (1-x)2/6 going down. My point is that I'll be able to figure out what that-- this is u, this is the u. You're going to say, why? Let me pause before putting in numbers and say why is it, how do I know that the solution is right on the function, the continuous solution.

Well, can I draw a set of pictures just like those guys for discrete? Yeah, let me just draw those for discrete here. That shows you the magic. So there is a-- I'm going to draw a vector now. I'm going to have to lift the chalk, it won't be a function and it'll be the delta vector. So it'll be the delta vector, delta with-- So there is point one, zero, one, two, up to six. It'll be the delta vector. Well if I just draw the delta vector, the delta vector has a one there. So this is the delta vector. Do I need? Well you can see that the delta vector is now going to be the vector of all zeroes and it's got a one at the key-- at the impulse and then zero. So it's a discrete impulse. That would be a better word. Discrete impulse. Impulse at zero. So let's stay with an impulse at zero.

Alright. What's my next picture? Again let me put in zero. One, two, three, onwards. Minus one, so on. What do I want to do now? What do I draw second? I always look over here. What did I draw second over here? The step. Now why did I draw a step function? How did I get from here to here? I integrate. I took the integral. So how will I get from here to this picture? I don't integrate, I add, sum. So coming along from the left, all these all along here, this sum is all zero because it was always zero. So it's zero, zero, zero, zero. And then, whoops, wait a minute. It says it a one there? Yeah, I think it must be. So here it wasn't a zero, wrong. Here it's a one. And what is it next? What's next to it? One, because I'm adding more and more zeroes but I have that one now, okay. A discrete step. It's a discrete step, zeroes and then ones. Now comes the second. So what am I going to call that? A step, right? It'll be a step function, step vector. If the sums of the delta vector gave me the step vector, how do I go the other way? What do I do to the step vector to get back to the delta vector? Differences, right? Sums in one direction, differences in the other. So the differences of the step vector are the delta vector. The step is the sum of the deltas and the delta is the differences of the step.

Now for the crucial next guy. What's it going to be? I add. Wait a minute. What's up? I'm looking for that picture. Do I get it? Yeah, I hope so. Oh, look, we ran out of time. I don't have to do this, but I will. So as I add I get zeroes and then it's one, and then I add on one more one. Look. You see what's happening. I run along at zero but I'm going to look at the book to see whether that jump should come here or here. So I've got a little bit of this to finish next time and I'm open for any questions this afternoon. Okay, thanks and sorry to keep you late.