Corrections and Changes to the First Printing

Revised July 29, 2004

The first printing has 10 9 8 7 6 5 4 3 2 1 on the first left-hand page.

Bullets mark the more significant changes or corrections.

- p. 10, Def. 1.6B: *read:* Any such $C\ldots$
- • p. 12, Ex. 1.3/1: *add:* (d) $\sum_0^n \sin^2 k\pi/2$
- • p. 30, Ex. 2.1/3: *replace:* change the hypothesis on $\{b_n\}$ *by:* strengthen the hypotheses (cf. p. 405, Example A.1E for the meaning of "stronger")
- p. 31, Ex. 2.4/5: *replace:* c_i *by* c_k
- p. 31, Ex. 2.5/1: *delete second* ϵ: $a^n \approx b^n$
- • p. 32, Ex. 2.6/4: *replace by:* Prove $\{a_n\}$ is decreasing for $n \gg 1$, if $a_0 = 1$ and

$$\text{(a)}\quad a_{n+1} = \frac{n-5}{(n+1)(n+2)}\,a_n \qquad \text{(b)}\quad a_{n+1} = \frac{n^2+15}{(n+1)(n+2)}\,a_n$$

- • p. 32, Prob. 2-4: *replace by:*

A positive sequence is defined by $a_{n+1} = \sqrt{1 + a_n^2/4},\ 0 \le a_0 < 2/\sqrt{3}$.

(a) Prove the sequence is strictly increasing.

(b) Prove the sequence is bounded above.

- p. 47, Ex. 3.3/1d: *delete the semicolons*
- • p. 48, *add two problems:*

3-4 Prove that a convergent sequence $\{a_n\}$ is bounded.

3-5 Given any c in $\mathbf{R}$, prove there is a strictly increasing sequence $\{a_n\}$ and a strictly decreasing sequence $\{b_n\}$, both of which converge to c, and such that all the a_n and b_n are (i) rational numbers; (ii) irrational numbers. (Theorem 2.5 is helpful.)

- p. 52, Example 4.2, Solution: *change:* $-u$ for u *to:* $-u$ for a.
- p. 55, line 11: *read:* e_0^4
- • p. 58, Ex. 4.3/2: *Omit.* (too hard)
- • p. 58, Ex. 4.4/1: *replace the last line by:*

Guess what its limit L is (try an example; cf. (15), 4.4). Then by finding the recursion formula for the error term e_n, prove that the sequence converges to L

(a) if $A > B$; (b) if $A < B$.

- • p. 59, Ex. 4.4/3: *replace (b) and (c) by:*

(b) Show that the limit is in general not 1/2 by proving that

(i) $a_0 < 1/2 \Rightarrow \lim a_n = 0$; (ii) $a_0 > 1/2 \Rightarrow \lim a_n = \infty$.

- • p. 59, Prob. 4-2b: *add:*

Use the estimations $|1 - \cos x| \le x^2/2$ and $|\sin x| \le |x|$, valid for all x.

- p. 60, Ans. 4.3/2: *read:* 1024
- p. 63, line 11 from bottom: *read:* 5.1/4
- p. 63, display (9): *delete:* > 0
- p. 67, line 1: *read:* Example 5.2C
- p. 68, line 10: *replace:* hypotheses *by:* symbols
- • p. 74, Ex. 5.3/4a: *add:* (Use Problem 3-4.) *(see above on this list)*
- • p. 74, Ex. 5.4/1 *Add two preliminary warm-up exercises:*

a) Prove the theorem if $k = 2$, and the two subsequences are the sequence of odd terms a_{2i+1}, and the sequence of even terms a_{2i}.

b) Prove it in general if $k = 2$.

c) Prove it for any $k \ge 2$.

p. 74, Ex. 5.4/2: *add:* (Use Exercise 3.4/4.)

- p. 75, Prob. 5-1(a): *replace the first line of the "proof" by:*
  Let $\sqrt{a_n} \to M$. Then by the Product Theorem for limits, $a_n \to M^2$, so that

p. 82, Proof (line 2): *change:* a_n *to* x_n

p. 89, Ex. 6.1/1a: *change* c_n *to* a_n

- p. 89, Ex. 6.1/1b: *add:* Assume $b_n - a_n \to 0$.
  add at end: to the limit L given in the Nested Intervals Theorem.
- p.89, Ex. 6.2/1: *make two exercises (a) and (b), and clarify the grammar:*
  Find the cluster points of: (a) $\{\sin(\frac{n+1}{n}\frac{\pi}{2})\}$ (b) $\{\sin(n+\frac{1}{n})\frac{\pi}{2}\}$.
  For each cluster point, find a subsequence converging to it.
- p. 89, Ex. 6.2/2: *replace by following exercise:*
  The terms of a sequence $\{x_n\}$ take on only finitely many values $a_1, \ldots, a_k$.
  That is, for every n, $x_n = a_i$ for some i (the index i depends on n.)
  Prove that $\{x_n\}$ has a cluster point.
- p. 89, bottom, *add:* **3.** Find the cluster points of the sequence $\{\nu(n)\}$ of Problem 5-4.
- p. 90, *add Exercise 6.3/2:* **2.** Prove the Bolzano-Weierstrass Theorem without using the Cluster Point Theorem (show you can pick an x_{n_i} $[a_i, b_i]$).

p. 90, Ex. 6.5/4: *read:* non-empty bounded subsets

p. 95, Display (6): *delete:* e

p. 104, Display (12). *change:* $f(n)$ *to* $f(n+1)$.

p. 106, l. 10 *read:* $-\sum(-1)^n a_n$

p. 107, l. 2,3 *insert:* this follows by Exercise 6.1/1b, or reasoning directly, the picture

p. 108, bottom half of the page *replace everywhere:* "positive" and "negative" by "non-negative" and "non-positive" respectively

p. 124, Ex. 8.4/1 *read:* $\sum_0^\infty$

- p. 135, Prob. 9-1. *delete:* for all x; *add at end:* on that interval.
- p. 135, Prob. 9-2: *replace last two sentences by:*
  Show the analogous statement for $x > 0$ and a strictly *decreasing* function is false.
- p. 148, Ex. 10.1/7a(ii) *read:* is strictly decreasing

p. 149, Ex. 10.3/5: *renumber:* 10.3/4

p. 154, line 8 from bottom *insert paragraph:*

On the other hand, functions like the one in Exercise 11.5/4 which are discontinuous (i.e., not continuous) at every point of some interval are somewhat pathological and not generally useful in applications; in this book we won't refer to their x-values as points of discontinuity since "when everybody's somebody, then no one's anybody".

p. 161, line 11: *delete ;,* line 12: *read* $<$, line 13 *read* $\leq$

p. 164, *read:*

Theorem 11.4D' Let $x = g(t)$, and I and J be intervals. Then

$g(t)$ continuous on I, $g(I) \subseteq J$, $f(x)$ continuous on $J \Rightarrow f(g(t))$ continuous on I.

- p. 166, Theorem 11.5A: *read:* $\lim_{x\to a} f(x) = L$. *read:* $x_n \to a,\ x_n \neq a$
- p. 166, Theorem 11.5B: *read:* $x_n \to a,\ x_n \neq a$
- p. 167, Ex. 11.1/3: *add:* (Use $|\sin u| \leq |u|$ for all u.)

p. 167, Ex. 11.1/4 *read:* exponential law, $e^{a+b} = e^a e^b$,

- p. 167, Ex. 11.2/2: *rewrite:* Let $f(x)$ be even; prove: $\lim_{x\to 0^+} f(x) = L \Rightarrow \lim_{x\to 0} f(x) = L$.

p. 167, Ex. 11.3/1a: *add:* $x \neq 0$

- p. 167, Ex. 11.3/1b: *read after semicolon:* using one of the preceding exercises.

p. 168, Ex. 11.3/6: *add:* $n > 0$

- p. 168, Ex. 11.5/2: *rewrite:* Prove $\lim\limits_{x\to\infty} \sin x$ does not exist by using Theorem 11.5A.
- p. 169, Prob. 11-2: *read:* a positive number c ...

  p. 180, Ex. 12.1/3: *read:* a polynomial

  p. 181, Ex. 12.2/3: *change:* solutions *to* zeros
- p. 182, Prob. 12-1: *replace:* Theorems 11.3C and 11.5 *by* Theorem 11.4B
- p. 188, Ques. 13.3/3: *read:* $(0, 1]$
- p. 192, Ex. 13.1/2 *renumber as 13.2/2, and change part (b) to:*

  13.2/2b Prove the function of part (a) cannot be continuous.
- p. 192, Ex. 13.3/1: *read:* $\lim f(x) = 0$ as $x \to \pm\infty$

  p. 193, Ex. 13.4/2: *read:* italicized property on line 2 of the ...

  p. 193, Ex. 13.5/2 *change the two* R *to* $\mathbf{R}$

  p. 194, Prob. 13-5: *read:* 13.4/1
- p. 195, Ans. 13.3/3: *change to:* $\frac{1}{x}\sin(\frac{1}{x})$; as $x \to 0^+$, it oscillates ever more widely

  p. 204, line 4: *read:* an open I
- p. 208, line 13: *replace by:* then show this limit is 0 and finish the argument using (b).

  p. 208, line 19: *change* fourteen *to* several
- p. 218, Ex. 15.2/2b: *read:* $0 < a < 1$
- p. 219, Ex. 15.3/2b: *read:* Prove (15) by applying the Mean-Value Theorem to

  $$F(t) = f(t)\big(g(b) - g(a)\big) - g(t)\big(f(b) - f(a)\big)$$
- p. 219, Prob. 15-2: *change to:*

  show that between two zeros of f is a zero of g, and vice-versa

  p. 228, Ex. 16.1/1a,b: *read:* $(0, 1]$

  p. 228, Ex. 16.1/1c: *renumber:* 1b
- p. 228, Ex. 16.1/3: *read:* $a \in [0, 2]$
- p. 228, Ex. 16.2/1: *read:* converse of each statement in (8) is not true
- p. 229, Prob. 16-1: *read:*

  Prove: on an an open interval I, a geometrically convex function $f(x)$ is continuous. (Show $\lim\limits_{\Delta x\to 0^-} \Delta y/\Delta x$ exists at each point of I; deduce $\lim\limits_{\Delta x\to 0^-} \Delta y = 0$.)

  p. 230, Ans. 16.1/2 *change 9 to 0*

  p. 231, line 3- *change k to a*

  p. 235, display (15): *change* $0 < |x| < |x|$ *to* $\begin{cases} 0 < c < x, \\ x < c < 0\ . \end{cases}$; *delete next two lines*
- p. 239, Ex. 17.4/1c: *add:* for $-1 < x \le 0$
- p. 239, Ex. 17.4/1d: *add:* for $0 \le x < 1$

  p. 243, Example 18.2, Solution, lines 4 and 7 *read:* $[0, x_1]$
- p. 248, Ex. 18.2/1 *add:* Hint: cf. Question 18.2/4; use $x_i^2 - x_{i-1}^2 = (x_i + x_{i-1})(x_i - x_{i-1})$.

  p. 248, Ex. 18.3/1 *replace n by k everywhere*

  p. 260, Defn. 19.6 *read:* $a = x_0 < x_1 < \ldots < x_{n-1} < x_n = b$

  p. 261, Solution. (b) *read:* $[1/(n + 1)\pi, 1/n\pi]$

  p. 261, Lemma 19.6 *rename: Endpoint Lemma*

  p. 261, line 7- *replace:* $[c, d]$ *by* $[a, b]$
- p. 263, Ex. 19.2/2: *read:* lower sums,
- p. 264, Ex. 19.4/3: *change:* $\ln(6.6)$ *to* $\pi/10$
- p. 265, Ex. 19.6/1: *call the statement in ( ) part (b), and in its line 2, replace* $f(x)$ *by* $p(x)$
- p. 266, Prob. 19-2: *make the "Prove" statement part (a), then add:*

  (b) Prove the converse: if $f(x)$ on $[a, b]$ is integrable with integral $\mathcal{I}$ in the sense of the above definition, then it is integrable and its integral is $\mathcal{I}$ in the sense of definitions 18.2 and 19.2. (Not as easy as (a).)

- p. 284, Ex. 20.3/5b: *change to:*

  (b) In the picture, label the u-interval $[a_1, x]$ and the v-interval $[a_2, y]$.

  If a continuous strictly increasing elementary function $v = f(u)$ has an antiderivative that is an elementary function, the same will be true for its inverse function $u = g(v)$ (which is also continuous and strictly increasing, by Theorem 12.4).

  Explain how the picture shows this.
- p. 284, Ex. 20.5/2: *read:* give an estimate $f(n)$ for the sum C_n and prove it is correct to within 1.

p. 287, Prob. 20-6b: *change: 11.3B to 5.2*

p. 289, Ans. 20.5/1: *read:* 1024

p. 294, line 6 from bottom: integral on the right is $\int_{a^+}^{b} g(x)\,dx$

p. 300, Ex. 21.2/2: *delete on second line:* dx

- p. 301, Prob. 21-3: *delete hint, add hypothesis:* $\int_a^\infty f'(x)\,dx$ is absolutely convergent.

p. 307, Example 22.1C *read:* Show: as $n \to \infty$, $\dfrac{n}{1+nx} \dots$

p. 310, Theorem 22.B *read:* $\sum_0^\infty M_k$

p. 311, line 3 from bottom: *change 4 to 3b*

p. 322, Ex. 22.1/3 *read:* $u_k(x) =$

p. 331, line 3: *change 21.1c to 23.1Ac*

p. 332, middle *delete both* $\aleph_1$, *replace the second by* $N(\mathbf{R})$

p. 344, Prob. 23-1 hint: *change* continuities *to* discontinuities

p. 357, Theorem 24.7B, line 2 *read:* non-empty compact set S; line 6 *read:* bounded and non-empty;

- p. 359, Ex. 24.1/3: all x should be in boldface type
- p. 361, Ex. 24.7/2: *read:* two distinct points not in S. Prove there is an x in S which...
- p. 384, Prob. 26-2: *add:* Assume the y_i have continuous second derivatives.

p. 385, line 2- *read:* $\int_0^1$ (cf. Theorem D.3A for the denominator.)

p. 391, Th. 27.4A line 2: *replace* I *by* $[a, b]$

p. 396, Ans. 27.2/2c, line 2: *read:* $te^{-t} < e^{(e-1)t}$

p. 404, Example A.1C(i): *read:* $a^2 + b^2 = c^2$

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