18.100**C**
Fall 2012

Problem Set 9

1. Recall that $\mathcal{C}^1([a,b])$ is defined as the space of functions $f : [a,b] \to \mathbb{R}$ which are everywhere differentiable and whose derivative f' is a bounded function. One equips this space with the metric

$$d(f,g) = \sup\{|f(x) - g(x)|\} + \sup\{|f'(x) - g'(x)|\}.$$

   Prove that this turns it into a *complete* metric space. **Please write up your proof carefully in LaTeX.** (3 points)

2. Let (f_n) be a sequence of functions $[a,b] \to \mathbb{R}$. Suppose that each f_n is a step function, and that they uniformly converge to a function f. Show that for each p, the one-sided limits

$$\lim_{x \to p-} f(x), \quad \lim_{x \to p+} f(x)$$

   exist (one-sided limits are defined in the book on page 94, where they are called $f(p-)$ and $f(p+)$; you may use either notation). (5 points)

3. Let $f : [-1,1] \to \mathbb{R}$ be a continuous function which is odd, $f(x) = -f(-x)$. Show that then, there is a sequence of polynomials which are odd and which uniformly converge to f. (4 points)

Total: 3+5+4 = 12 points.