18.100C Lecture 11 Summary

Definition of power series. Convergence radius ρ of a power series.

Theorem 11.1. *$f(z) = \sum_{k=0}^{\infty} a_k z^k$ is absolutely convergent for all complex numbers $|z| < \rho$.*

The series never converges for $|z| > \rho$. However, for $|z| = \rho$ several types of behaviour are possible.

Theorem 11.2. *Take a series $f(z) = \sum_{k=0}^{\infty} a_k z^k$, where $a_k \in \mathbb{R}$ and $a_0 \geq a_1 \geq a_2 \geq \cdots$, $\lim_{k\to\infty} a_k = 0$. Suppose that the convergence radius is 1. Then the series converges for all z such that $|z| = 1$ and $z \neq 1$.*

Theorem 11.3 (Abel; not proved in class)**.** *Take a series $f(z) = \sum_{k=0}^{\infty} a_k z^k$ with $a_k \in \mathbb{R}$. Suppose that $\sum_k a_k$ is convergent. Then its value is $\lim_{t\to 1} f(t)$, where the limit is taken over real $t < 1$.*

The exponential series $\exp(z)$. It has infinite convergence radius (converges absolutely for all $z \in \mathbb{C}$).

Theorem 11.4. $\exp(z)\exp(w) = \exp(z+w)$.

Theorem 11.5. $|\exp(z)| = \exp(Re(z))$.

Definition of sin and cos by $\exp(it) = \cos(t) + i\sin(t)$. Power series for cos and sin.

Theorem 11.6. $\cos^2(t) + \sin^2(t) = 1$.

The trigonometric addition formulae.
Short discussion of Fourier series.