18.100C Lecture 13 Summary

Example 13.1. *The map* $f : \mathbb{R}^2 \to \mathbb{R}$, $f(x, y) = xy$, *is continuous.*

Example 13.2. *The map* $\exp : \mathbb{R} \to \mathbb{R}$ *is continuous (the same holds for* $\exp$ *of a complex number, hence also for* $\cos$ *and* $\sin$*).*

Theorem 13.3. *(Intermediate Value Theorem) Let* $f : [a, b] \to \mathbb{R}$ *be a continuous map, such that* $f(a) \leq 0$, $f(b) \geq 0$. *Then there is some* x *such that* $f(x) = 0$.

Corollary 13.4. *Let* $f : [a, b] \to \mathbb{R}$ *be a continuous map. Then its image is a closed interval* $[c, d]$.

Corollary 13.5. *Let* $f : [a, b] \to \mathbb{R}$ *be a continuous map which is strictly increasing (*$x < y$ *implies* $f(x) < f(y)$*). Then* f *is one-to-one onto a closed interval* $[c, d]$. *Moreover, the inverse map* $f^{-1} : [c, d] \to [a, b]$ *is continuous.*

Example 13.6. *The map* $\exp$, *from the real numbers to the positive real numbers, is strictly increasing and onto. We call its inverse the natural logarithm* $\log : (0, \infty) \to \mathbb{R}$. *This automatically satisfies* $\log(ab) = \log(a) + \log(b)$, *and is continuous.*

Let (X, d_X) and (Y, d_Y) be metric spaces, and $f : X \to Y$ a map.

Definition 13.7. f *is uniformly continuous if: for any* $\epsilon > 0$ *there is a* $\delta > 0$ *such that if* $d_X(x, y) < \delta$, *then* $d_Y(f(x), f(y)) < \epsilon$.

Every absolutely continuous map is continuous.

Theorem 13.8. *If* X *is compact, every continuous map* $f : X \to Y$ *is uniformly continuous.*

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