18.100C Lecture 16 Summary

Pointwise convergence. Examples. Uniform convergence.

Theorem 16.1 (Cauchy convergence criterion)**.** *A sequence of functions $f_n : X \to \mathbb{R}$ is uniformly convergent if and only if the following holds. For every $\epsilon > 0$ there is an N such that if $m, n \geq N$ then $|f_n(x) - f_m(x)| < \epsilon$ for all x.*

Uniform convergence of series of functions.

Corollary 16.2. *(Weierstrass criterion) Let $\sum_{n=0}^{\infty} f_n$ be a series of functions. Suppose that there are constants M_n such that $|f_n(x)| \leq M_n$ for all n, x, and such that $\sum_{n=0}^{\infty} M_n$ converges. Then $\sum_{n=0}^{\infty} f_n$ converges uniformly.*

Corollary 16.3. *Let $\sum_{n=0}^{\infty} a_n x^n$ be a power series with radius of convergence $\rho > 0$. Then that series converges uniformly on any interval $[-r, r]$ with $r < \rho$.*

Theorem 16.4. *If (f_n) are continuous functions converging uniformly towards f, then f is again continuous.*

Corollary 16.5. *Let $f(x) = \sum_{n=0}^{\infty} a_n x^n$ be a power series with radius of convergence $\rho > 0$. Then f is continuous on $(-\rho, \rho)$.*

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