18.100C Lecture 5 Summary

Throughout, (X, d) is an arbitrary metric space.

Theorem 5.1. *If $E, F \subset X$ are open subsets, then so are $E \cup F$ and $E \cap F$.*

Theorem 5.2. *If (E_i) is a collection of open subsets of X indexed by $i \in I$ for some set I, then their union $\bigcup_{i \in I} E_i$ is also open.*

Corollary 5.3. *Every open subset is a union of ball neighbourhoods.*

Definition of limit point, closed subset.

Theorem 5.4. *If x is a limit point of E, then $B_r(x) \cap E$ is infinite for any $r > 0$.*

Corollary 5.5. *A finite subset of X has no limit points, hence is closed.*

Theorem 5.6. *If $E, F \subset X$ are closed subsets, then so are $E \cup F$ and $E \cap F$.*

Theorem 5.7. *If (E_i) is a collection of closed subsets of X indexed by $i \in I$ for some set I, then their intersection $\bigcap_{i \in I} E_i$ is also closed.*

Theorem 5.8. *A subset $E \subset X$ is open if and only if its complement $X \setminus E$ is closed.*

Definition of closure $\bar{E}$.

Definition 5.9. *A subset $E \subset X$ is called dense if $\bar{E} = X$.*

18.100C Real Analysis
Fall 2012