18.100C Lecture 7 Summary

Theorem 7.1. *Let (X, d) be a metric space with the following property: every countably infinite subset $E \subset X$ has a limit point. Then X is compact.*

Step 1: show that X has an at most countable dense subset (homework).
Step 2: show that if $(U_i)_{i \in I}$ is an open cover of X, then at most countably many U_i already cover X.
Step 3: show that if $(U_i)_{i \in I}$ is a countable open cover of X, then finitely many U_i already cover X.

Theorem 7.2 (Heine-Borel)**.** *Every finite closed interval $[a, b] \subset \mathbb{R}$ is compact (for the standard metric).*

Theorem 7.3. *Every bounded closed subset of $\mathbb{R}$ is compact.*

Theorem 7.4. *Every finite closed cube $[a_1, b_1] \times \cdots \times [a_n, b_n] \subset \mathbb{R}^n$ is compact.*

Theorem 7.5. *Every bounded closed subset of $\mathbb{R}^n$ is compact.*