18.100C Lecture 8 Summary

Convergent sequences in metric spaces. Examples.

Theorem 8.1. *Let (x_n) be a convergent sequence, where all the x_n lie in a subset $E \subset X$. Then the limit x lies in $\bar{E}$.*

Theorem 8.2. *If $x \in \bar{E}$, there is a sequence (x_n), $x_n \in E$, which converges to x.*

Subsequence of a convergent sequence is convergent (same limit).

Theorem 8.3. *Let (X, d) be a compact metric space. Then every sequence (x_n) in X has a convergent subsequence.*

Corollary 8.4. *Every bounded sequence in $\mathbb{R}^d$ has a convergent subsequence.*

Definition of Cauchy sequence. Every convergent sequence is a Cauchy sequence.

Lemma 8.5. *Let (x_n) be a Cauchy sequence. If it has a convergent subsequence, then (x_n) itself converges (to the same point).*

Theorem 8.6. *Let (X, d) be a compact metric space. Then every Cauchy sequence converges.*

Corollary 8.7. *Every Cauchy sequence in $\mathbb{R}^n$ converges.*

A metric space where this happens (every Cauchy sequence converges) is called complete. So, we just showed that compact metric spaces as well as $\mathbb{R}^n$ are complete.

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