18.100C Lecture 9 Summary

Subsequential limits (accumulation points) of a sequence in a metric space.

Theorem 9.1. *The set of accumulation points of any sequence is a closed subset.*

Theorem 9.2. *Suppose that the metric space (X, d) is separable (has a countable dense subset). Then for every closed nonempty subset $E \subset X$ there is a sequence (x_n) whose set of accumulation points is precisely E. [No proof in class]*

Convergence of sequences in $\mathbb{R}$.

Theorem 9.3. *Let (x_n) be a sequence which is nondecreasing, $x_1 \leq x_2 \leq x_3 \cdots$. Then (x_n) converges if and only if it is bounded above.*

Theorem 9.4. $x_n = (1 + 1/n)^n$ *converges.*

Definition of $\limsup$ and $\liminf$. Improper limits $\pm\infty$.
Convergence of series. Series of nonnegative numbers.

Theorem 9.5. *A series of nonnegative numbers converges if and only if its partial sums are bounded above.*

Theorem 9.6. $\sum_{k=0}^{\infty} x^p = 1/(1-x)$ *for all* $|x| < 1$.

Theorem 9.7. $\sum_{k=1}^{\infty} 1/k^p$ *diverges if* $p \leq 1$, *and converges if* $p > 1$.