MATH 18.152 - PROBLEM SET 4

18.152 Introduction to PDEs, Fall 2011 — Professor: Jared Speck

Problem Set 4, **Due: at the start of class on 10-6-11**

I. Problem **3.1** on pg. 150.
II. Problem **3.2** on pg. 150.
III. Problem **3.3** on pg. 151. You may assume that $u \in C^3(\Omega) \cap C^1(\overline{\Omega})$.
IV. Problem **3.4** on pg. 151.
V. Problem **3.8** on pg. 152.
VI. Let $B_1(0)$ denote the solid unit ball in $\mathbb{R}^n$, and let $\partial B_1(0)$ denote its boundary. Let $f(x)$ be smooth (i.e., infinitely differentiable) function on $B_1(0)$, let $g(\sigma)$ be a smooth function on $\partial B_1(0)$, and let $u(x)$ be a smooth solution to

$$\begin{aligned} \Delta u(x) &= f(x), \qquad x \in B_1(0), \\ u(\sigma) &= g(\sigma), \qquad \sigma \in \partial B_1(0). \end{aligned}$$

Show that there exists a constant $C > 0$ which does *not* depend on f or g such that

$$\max_{x \in B_1(0)} |u(x)| \leq C\big(\max_{x \in B_1(0)} |f(x)| + \max_{\sigma \in \partial B_1(0)} |g(\sigma)|\big).$$

If you prefer, you can supply a proof for the case $n = 3$ only (the remaining cases are similar).
Hint: Revisit the proof of the Mean value properties discussed in class.

18.152 Introduction to Partial Differential Equations.
Fall 2011