Lecture 13

In order to get an intuitive feel for what the eigenfunctions should look like, a powerful tool is the **min–max theorem**. See handout for notes.

As a final example corresponding to the $-c\nabla^2$ operator in the notes, considered an "L"-shaped domain Ω with c=1/w(x). In particular, suppose that w(x)=1 everywhere except for a small region where $w(x)=w_0>1$. In order to concentrate in this small region, u(x) will have to have bigger slope (sacrificing the numerator). As w_0 increases, we expect the denominator of the Rayleigh quotient to "win" and the concentration to increase, while for w_0 close to 1 the eigenfunctions should be similar to the case of $-\nabla^2$.

Further reading: See, for example min-max theorem in Wikipedia, although this presentation is rather formal. Unfortunately, most of the discussion you will find of this principle online and in textbooks is either (a) full of formal functional analysis or (b) specific to quantum mechanics [where the operator is $A=-\nabla^2+V$ for some "potential-energy" function V(x)].

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