## Lecture 2

Started with a very simple vector space V of functions: functions u(x) on [0,L] with u(0)=u(L)=0 (Dirichlet boundary conditions), and with one of the simplest operators: the 1d Laplacian $\hat{A}=d^2/dx^2$. Explained how this describes some simple problems like a stretched string, 1d electrostatic problems, and heat flow between two reservoirs.

Inspired by 18.06, we begin by asking what the null space of $\hat{A}$ is, and we quickly see that it is {0}. Thus, any solution to $\hat{A}u=f$ must be unique. We then ask what the eigenfunctions are, and quickly see that they are $\sin(n\pi x/L)$ with eigenvalues $-(n\pi/L)^2$. If we can expand functions in this basis, then we can treat $\hat{A}$ as a number, just like in 18.06, and solve lots of problems easily. Such an expansion is precisely a Fourier sine series (see handout).

In terms of sine series for f(x), solve $\hat{A}u=f$ (Poisson's equation) and $\hat{A}u=\partial u/\partial t$ with $u(x,0)=f(x)$ (heat equation). In the latter case, we immediately see that the solutions are decaying, and that the high-frequency terms decay faster...eventually, no matter how complicated the initial condition, it will eventually be dominated by the smallest-n nonzero term in the series (usually n=1). Physically, diffusion processes like this smooth out oscillations, and nonuniformities eventually decay away. Sketched what the solution looks like in a typical case.

As a preview of things to come later, by a simple change to the time-dependence found a solution to the wave equation $\hat{A}u=\partial^2 u/\partial t^2$ from the same sine series, which gives "wavelike" behavior. (This is an instance of what we will later call a "separation of variables" technique.)

**Further reading:** Section 4.1 of the Strang book (Fourier series and solutions to the heat equation).