Lecture 3

Now, we will go back to the happy land of finite-ness for a while, by learning to approximate a PDE by a matrix. This will not only give us a way to compute things we cannot solve by hand, but it will also give us a different perspective on certain properties of the solutions that may make certain abstract concepts of the PDE clearer. We begin with one of the simplest numerical methods: we replace the continuous space by a grid, the function by the values on a grid, and derivatives by differences on the grid. This is called a **finite-difference method**.

Went over the basic concepts and accuracy of approximating derivatives by differences; see handout.

Armed with center differences (see handout), went about approximating the 1d Laplacian operator d^2/dx^2 by a matrix, resulting in a famous tridiagonal matrix known as a *discrete Laplacian*. The properties of this matrix will mirror many properties of the underlying PDE, but in a more familiar context. We already see by inspection that it is real-symmetric, and hence we must have real eigenvalues, diagonalizability, and orthogonal eigenvectors—much as we observed for the d^2/dx^2 operator—and in the next lecture we will show that the eigenvalues are negative, i.e. that the matrix is negative-definite.

The negative eigenvalues mean that the discrete Laplacian is negative definite, and also suggest that it can be written in the form $-D^TD$ for some D. Reviewed the proof that this means the matrix is negative definite, which also relies on D being full column rank.

Further reading: notes on finite-difference approximations from 18.330. See the matrix K section 1.1 ("Four special matrices") of the Strang book, and in general chapter 1 of that book.

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