Lecture 35

Evaluated the Galerkin discretization of $\hat{A}=d^2/dx^2$ with tent basis functions. Showed that, for a uniform grid, we recover our center-difference finite-difference matrix. For a nonuniform grid, we get a generalization. Analyzed the accuracy to show that the generalization is still second-order accurate if the resolution is changing "continuously" (i.e. if it is approaching a continuously varying resolution as N increases), but is only first-order accurate if the resolution jumps. This means that grid-generation methods for finite elements try to produce meshes where the elements change in size smoothly.

On the other hand, if we define accuracy in an "average" sense (e.g. the L2 norm of the error), then it turns out that we always have second-order accuracy even if there are jumps in resolution (although these may have large localized contributions to the error). For positive-definite operators $\hat{A}$, we will use the fact (from last lecture) that Galerkin methods minimize an $\hat{A}$-weighted norm of the error in $\tilde{u}$ in order to flesh out a more careful convergence analysis.

Discussed some of the general tradeoffs of complexity in finite-element vs. finite-difference methods: more sophistication is not always better, especially since computer time is usually much cheaper than programmer time.

Further reading: See the notes on finite-element methods from 16.920J/2.097J/6.339J. Some nice free/open-source software packages for finite-element calculations are FEniCS, deal.II, and libMesh.

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