

## Lecture 36

Sketch of convergence proof for the finite-element method based on bounding the error  $\|u - u_h\|_A$ : relate this error  $\|u - u_h\|_A$  via the min-max theorem for positive-definite  $\hat{A}$  (coercive bilinear forms). Then showed that the latter is minimized since  $\tilde{u}$  is an orthogonal projection, and finally bounded  $\|u - u_h\|_A$  above by  $\|\tilde{u} - u_h\|_A$  where  $\tilde{u}$  is a simple Lagrange interpolation of  $u$ . (Unfortunately, the error estimate produced by this procedure is a bit too conservative: for first-order elements, this gives a first-order upper bound, when in fact the error is typically second-order.)

Boundary conditions and the finite-element method: essential vs. natural boundary conditions. Dirichlet is essential (imposed by the function space, or via explicit constraint equations), while Neumann is natural (imposed by the weak form in the absence of an essential boundary condition).

**Further reading:** Lectures 2 and 7 of [these course notes](#) by Joseph Flaherty at RPI were pretty readable regarding convergence (see, in particular, section 2.6 of lecture 2). The book [Understanding and Implementing the Finite Element Method](#) by Mark S. Gockenbach has a nice discussion of boundary conditions in chapter 2.

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