Lecture 38

Discussed the impact of a different kind of algebraic structure on the solution of linear PDEs: symmetry. Looked at the specific examples of mirror symmetries, the symmetries of a square, and translational symmetry. Showed that a symmetry corresponds to a symmetry operator that commutes with Â and preserves the boundary conditions, and allows us to find simultaneous eigenfunctions of Â and the symmetry operator. For mirror symmetries, this leads to even/odd solutions, and for translational symmetry this leads to separable solutions with exp(ikz) exponentials in the invariant directions.

However, for more complicated symmetry groups with multiple "interacting" operations, looking for simultaneous eigenfunction tells us the truth but not the whole truth. To see how the symmetry operations relate to one another, we use group representation theory. Defined representations of a group and gave a few examples to suggest how they relate to eigenfunctions, although no proofs were given.

Further reading: The general subject of symmetry and linear PDEs leads to group theory (for the symmetry group) and group representation theory (to generalize the symmetry "eigenfunctions" to non-commutative groups). For a simple introduction similar to the one in class but applied to Maxwell's equations, see e.g. chapter 3 of our book. For a more complete treatment, see any book on the applications of group theory to physics; my favorite is this book by Inui but it is out of print; a classic with cheap reprints is this book by Tinkham. See this summary of the key definitions and theorems in representation theory for understanding the consequences of representations for eigenfunctions and linear PDEs in general.

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