18.306 Advanced Partial Differential Equations with Applications
Fall 2009

Lecture 13 2009 10 21 WED

TOPICS: Shallow water and higher order terms.
        Traveling waves, shocks, and the effects of dispersion.
        Solitons. Small dispersion limit.

Continue and finish material in lecture 12. In particular:
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Traveling wave solutions for KdV: $u_t + (0.5*u^2)_x = epsilon^2*u_{xxx}$.

Can write them exactly, but easier to do it with phase plane analysis.
   Periodic traveling waves and solitary waves.
   No shocks.

What happens in the epsilon small limit?
 Smooth I.V. should start evolving as $u_t + u*u_x = 0$, approximately.
 But this then produces short scales, and the term $epsilon^2*u_{xxx}$
kicks
 in (preventing multiple values). However, no shocks can form (there
are
 none in this equation). What one observes is that short wave
oscillations
 [wave-length O(epsilon)] are generated near the points where
 $u_t + u*u_x = 0$ would produce infinite derivatives. These oscillations
 propagate away from these points, and the region with fast variations
 in the solution grows with time. No easy fix for cases like this. The
 small scales cannot be ignored (and shoved into a discontinuity) as
 in the cases where shocks form.