

18.310 Homework 4

Due Wednesday October 2nd at 6PM

Instructions: Remember to submit a separate PDF for each question. Do not forget to include a list of your collaborators or to state that you worked on your own.

1. Let $(a_n)_{n \geq 0}$ be the sequence defined by $a_0 = 0$, $a_1 = 5$ and $a_n = a_{n-1} + 6a_{n-2}$ for $n \geq 2$. Find an explicit expression for a_n .
2. Given some $r \in \mathbb{R}$, consider the generating function $F(x) = \frac{1}{(1-rx)^2}$ corresponding to a sequence $(f_n)_{n \geq 0}$. Find an explicit expression for f_n .
3. A binary number consists of a sequence of 0's and 1's, such as 0100, 110001 or even the empty sequence \cdot containing no bits. Let F denote the set of all pairs (a, b) where a and b are binary numbers. Let d_n be the number of pairs in F having a total of n bits. Thus $d_0 = 1$ as it corresponds only to (\cdot, \cdot) , while $d_1 = 4$ as there are 4 pairs with a total of 1 bit: $(0, \cdot), (1, \cdot), (\cdot, 0), (\cdot, 1)$.
 - (a) Find the generating function for $(d_n)_{n \geq 0}$
 - (b) Derive from it an explicit formula for d_n .
 - (c) Explain how you could have derived this expression directly without considering its generating function.

MIT OpenCourseWare
<http://ocw.mit.edu>

18.310 Principles of Discrete Applied Mathematics
Fall 2013

For information about citing these materials or our Terms of Use, visit: <http://ocw.mit.edu/terms>.