Fall 2013

18.310 Homework 8

Due Monday October 28th at 6PM

Instructions: Remember to submit a separate PDF for each question. Do not forget to include a list of your collaborators or to state that you worked on your own.

1. What is the optimum solution of the following linear program:

$$\begin{array}{rl} \text{Max} & 5x_1 + 7x_2 + 9x_3 + 11x_4 + 13x_5 \\ \text{subject to:} & \\ & 15x_1 + 28x_2 + 18x_3 + 44x_4 + 65x_5 = 2010 \\ & x_i \geq 0 \qquad\qquad i = 1, \ldots, 5. \end{array}$$

   Write the dual linear program, and find an optimum solution to the dual. What are the optimum values to the primal and dual linear programs?

2. Solve the following LP using the simplex method, showing your tableau and choice of pivot clearly at each step, and writing the final answer and objective value clearly.

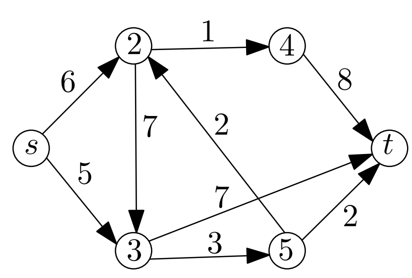
$$\begin{array}{rll} \text{maximize} & 3x_1 + 2x_2 + 4x_3 & \\ \text{subject to} & x_1 + x_2 + 2x_3 & \leq 4 \\ & 2x_1 \qquad + 3x_3 & \leq 5 \\ & 2x_1 + x_2 + 3x_3 & \leq 7 \\ & x_1, x_2, x_3 & \geq 0. \end{array}$$

   (You will need to introduce some slack variables to get the LP into the required form.)

3. Using strong duality for linear programming, prove the following.

   Theorem. *$Ax = b, x \geq 0$ has no solution if and only if there exists y with $A^T y \geq 0$ and $b^T y < 0$.*

4. Consider the following flow problem instance, with source s and sink t:



   Find a maximum flow (show your augmenting paths), and also exhibit an s-t cut of the same value.