EXAMPLE: River flow equations. $A_t + Q_x = 0$.

- Describe quasi-equilibrium function $Q = Q(A)$. dQ/dA increases with A. Justify: at equilibrium forces (friction and gravity) must balance.
- Note that $dq/d\rho$ (traffic flow) and dQ/dA (river floods) have the dimensions of a velocity. But, the velocity of what? We will answer this question soon.
- Compare $q = Q(\rho)$ and $q = Q(A)$ for traffic flow and river flow. Concave versus convex. In one case $c = dq/d\rho$ decreases and in the other increases.

EXAMPLE: Euler Equations of Gas Dynamics in 1-D.

Closure, quasi-equilibrium and thermodynamics. Polytropic gas.
IMPORTANT POINT: massive difference in adjustment time scales, as well as number of discrete elements used to compute the averages ===> Equations far more accurate in their predictions.
Body forces (sources).

Issue of SOURCES for conservation laws.
Examples:

- body forces in gas dynamics.
- feeder roads to main artery in traffic flow.
- feeder streams in river flows.

GENERAL FRAMEWORK. How conservation principles lead to pde's:
Integral and differential form of a conservation law.
Closure issues and "constitutive equations."
Quasi-equilibrium approximations and time-scale limitations.
The case of traffic flow and the traffic flow curve.

SEE/READ: conservation law notes on the WEB page.
Further examples will clarify these ideas:

- Small transverse vibrations of a string
- Small longitudinal vibrations of an elastic rod
- Heat flow along an insulated wire
- Heat flow in 2-D or 3-D
- Diffusion equation (e.g.: Salt in water)
- Viscosity in fluids
- Slow granular flow in a silo
- Gas Dynamics
- p-system
- String on an elastic bed
- Heat flow along an insulated wire.

Describe closure, Fick's law, and do intuitive justification using stat. mech. interpretation of heat and temperature.

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