Chapter 9

Optimization

9.1 Regularization and sparsity

9.2 Dimensionality reduction techniques

One way to reduce the dimensionality of a dataset is to scramble data as $\widetilde{d} = Cd$, where

$$\widetilde{d}_{j,r}(t) = \sum_s c_{j,s} d_{r,s}(t - b_{j,s}).$$

The numbers $c_{j,s}$ and $b_{j,s}$ may be random, for instance. The point is that using fewer values of j than s may result in computational savings — a strategy sometimes called source encoding. By linearity of the wave equation, the scrambled data $\widetilde{d}$ can be seen as originating from scrambled shots, or *supershots* $\widetilde{f} = Cf$, for

$$\widetilde{f}_j(x,t) = \sum_s c_{j,s} f_s(x, t - b_{j,s}).$$

Scrambled data may be all that's available in practice, in acquisition scenarios known as simultaneous sourcing.

The adjoint operation C^* results in twice-scrambled data $D = C^* \widetilde{d}$, where

$$D_{r,s}(t) = \sum_j c_{j,s} \widetilde{d}_{j,r}(t + b_{j,s}).$$

The linearized forward model with scrambling is $\widetilde{d} = CFm$. The basic imaging operator is still the adjoint, $I_m = F^* C^* \widetilde{d}$. In addition to the traditional incident and adjoint fields

$$u_{0,s} = G f_s, \qquad q_s = \overline{G} d_s,$$

where G is the Green's function in the unperturbed medium, and $\overline{G}$ the time-reversed Green's function, we define the scrambled fields

$$\widetilde{u}_{0,j} = G \widetilde{f}_j, \qquad \widetilde{q}_j = G \widetilde{d}_j.$$

Also define the twice-scrambled adjoint field

$$Q_s = G(C^* \widetilde{d})_s.$$

Then

$$I_m(x) = (F^* C^* \widetilde{d})(x) = -\sum_s \int_0^T \frac{\partial^2 u_{0,s}}{\partial t^2}(x,t)\, Q_s(x,t)\, dt.$$

Another formula involving j instead of s (hence computationally more favorable) is

$$I_m(x) = -\sum_j \int_0^T \frac{\partial^2 \widetilde{u}_{0,j}}{\partial t^2}(x,t)\, \widetilde{q}_j(x,t)\, dt. \tag{9.1}$$

To show this latter formula, use $Q = C^* \widetilde{q}$, pass C^* to the rest of the integrand with $\sum_s v_s (C^* w)_s = \sum_j (C v_j) w_j$, and combine $C u_0 = \widetilde{u}_0$.

Scrambled data can also be used as the basis of a least-squares misfit, such as

$$\widetilde{J}(m) = \frac{1}{2} \| \widetilde{d} - C \mathcal{F}(m) \|_2^2.$$

The gradient of $\widetilde{J}$ is $F^* C^*$ applied to the residual, hence can be computed with (9.1).

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