18.330 Brief Notes on **EXTRAPOLATION** Fri 05 Mar 04

Here is some experimental data from a rather dumb way to integrate

$$I = \int_0^1 (1 - x^4)^{1/4}\, dx$$

— or to estimate one-quarter of the area pursued in Problem 7 — using just the equispaced trapezoidal sums T_1, T_2, T_4, T_8, ... which are surely **not** ideal for this task.

But notice that even "junk" like this can be repaired or accelerated a lot via thoughtful extrapolations. Somewhat as in Problem 11a, these extrapolations begin with the insight or hunch that errors here shrink with increasing n about like $n^{-5/4}$. After making such repairs in a $2^{5/4} - 1$ fashion, it is time to pause and check the ratios of new differences to see if already from them we can spot the power law of the main errors remaining. Aha, these look like ...

n	T_n	$2^{5/4} - 1$ extrap	diff	ratio	$2^{9/4} - 1$ extrap
1	0.500000000				
		0.**9**17559606			
2	0.741997418		0.007471975		0.**9270**20486
		0.**925**031581		4.7068	
4	0.848075195		0.001587499		0.**9270**41643
		0.**92**6619080		4.8016	
8	0.893595444		0.000330617		0.**927037**701
		0.**92**6949696		4.7747	
16	0.**9**12925961		0.000069243		0.**9270373**71
		0.**9270**18940		4.7644	
32	0.**92**1093572		0.000014533		0.**9270373**42
		0.**92703**3473		4.7603	
64	0.**92**4536052		0.000003053		0.**927037339**
		0.**92703**6526		4.7585	
128	0.**92**5985207		0.000000642		0.**927037339**
		0.**927037**168		4.7576	
256	0.**92**6594873		0.000000135		0.**927037339**
		0.**927037**303			
512	0.**92**6851284				

Alternatively, one can conduct successive **Aitken extrapolations** on the above T_n data — repeating such heroics as soon as a fresh trio of numbers becomes available. Here is what we would have obtained then:

1	0.500000000				
2	0.741997418				
4	0.848075195	0.**9**30863045			
8	0.893595444	0.**927**812381			
16	0.**9**12925961	0.**927**193723	0.**92703**6348		
32	0.**92**1093572	0.**9270**69603	0.**92703**8452		
64	0.**92**4536052	0.**9270**44062	0.**927037**443	0.**927037**770	
128	0.**92**5985207	0.**92703**8746	0.**927037**349	0.**927037339**	
256	0.**92**6594873	0.**927037**634	0.**92703**340	0.**927037339**	0.**927037339**
512	0.**92**6851284	0.**927037**401	0.**927037339**	0.**927037339**	0.**927037339**