Bernoulli Polynomials

$$B_0(x) = 1$$

$$B_1(x) = x - \tfrac{1}{2}$$

$$B_2(x) = x^2 - x \boxed{+\frac{1}{6}} \equiv x(x-1) + \frac{1}{6}$$

$$B_3(x) = x^3 - \tfrac{3}{2}x^2 + \tfrac{1}{2}x \equiv x(x-\tfrac{1}{2})(x-1)$$

$$B_4(x) = x^4 - 2x^3 + x^2 \boxed{-\frac{1}{30}} \equiv x^2(x-1)^2 - \frac{1}{30}$$

$$B_5(x) = x^5 - \tfrac{5}{2}x^4 + \tfrac{5}{3}x^3 - \tfrac{1}{6}x \equiv x(x-\tfrac{1}{2})(x-1)\left[(x-\tfrac{1}{2})^2 - \tfrac{7}{12}\right]$$

$$B_6(x) = x^6 - 3x^5 + \tfrac{5}{2}x^4 - \tfrac{1}{2}x^2 \boxed{+\frac{1}{42}} \equiv x^2(x-1)^2\left[(x-\tfrac{1}{2})^2 - \tfrac{3}{4}\right] + \frac{1}{42}$$

$$B_7(x) = x^7 - \tfrac{7}{2}x^6 + \tfrac{7}{2}x^5 - \tfrac{7}{6}x^3 + \tfrac{1}{6}x$$

$$B_8(x) = x^8 - 4x^7 + \tfrac{14}{3}x^6 - \tfrac{7}{3}x^4 + \tfrac{2}{3}x^2 \boxed{-\frac{1}{30}}$$

As a rule, $dB_n/dx = nB_{n-1}(x)$ and $\int_0^1 B_n(x)dx = 0$ for all $n \geq 1$