

18.330 :: Homework 8 :: Spring 2012 :: Not due

Fourier exercises (05/15):

1. Show that a C^p periodic function ($p \geq 1$) has Fourier series coefficients that decay like $|k|^{-p}$.
2. Show that the error of the trapezoidal rule for integrating a C^p periodic function is $O(h^p)$.
3. Show that the L^2 error of bandlimited differentiation of a C^p periodic function is $O(h^{p-3/2})$.
4. Let $f(x) = \frac{1}{1+x^2}$, for which $\hat{f}(k) = \pi e^{-|k|}$. For the purpose of this exercise, assume $x \in \mathbb{R}$ and $k \in \mathbb{R}$ (no boundaries.)
 - Find the decay rate (as a function of N) of the error $\|f - f_N\|_2$ of best approximation by a function f_N with Fourier transform supported in $[-N, N]$.
 - Find the decay rate (as a function of h) of the error $\|f - p\|_2$ or bandlimited interpolation from the samples on the grid $x_j = jh$.
 - Same question for the error of bandlimited differentiation, and the error of the trapezoidal rule.
5. Formulate and prove the discrete convolution theorem.

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