Chapter 9

9.1 Computing Eigenvalues

$$Ax = \lambda x \tag{9.1}$$
$$A = X\Lambda X^{-1} \tag{9.2}$$
$$AX = X\Lambda \tag{9.3}$$

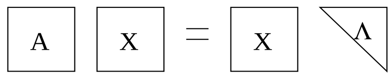

Figure 9.1: $AX = X\Lambda$.

$$Ax = \lambda x \Leftrightarrow (A - \lambda I)x = 0 \Leftrightarrow \det(A - \lambda I) = 0 \tag{9.4}$$

Definition: $P(\lambda) = \det(A - \lambda I)$. P is a polynomial of degree n.

Algebraic multiplicity as a root of P and geometric multiplicity equals to the number of linearly independent eigenvectors corresponding to λ.

Theorem: A and $X^{-1}AX$ have the same characteristic polynomial, eigenvalues, algebraic and geometric multiplicities.

Proof: $P(X^{-1}AX)$: OK. If E_λ eigenspace of A then $X^{-1}E_\lambda$ is an eigenspace for $X^{-1}AX$. Defective eigenvalue algebraic multiplicity and geometric multiplicity.

A matrix is called diagonalizable if it has an eigenvalue decomposition $A = X\Lambda X^{-1}$, where Λ is diagonal.

Unitarily diagonalizable:

$$A = Q\Lambda Q^* \tag{9.5}$$

happens when A is normal, i.e.,

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$$A^*A = AA^*. \tag{9.6}$$

9.2 Schur Factorization

$$A = QTQ^* \tag{9.7}$$

where, T-uppet triangular.

Theorem: Every matrix has a Schur factorization.

Algorithms: iteration

Scalar factorization and diagonalization: two phases.