3.7 Givens rotations

$$\begin{bmatrix} c & s \\ -s & c \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} \sqrt{x^2+y^2} \\ 0 \end{bmatrix} \tag{3.20}$$

Therefore,

$$cx + sy = \sqrt{x^2+y^2}, \tag{3.21}$$

$$sx = cy. \tag{3.22}$$

Finally,

$$c = \frac{y}{\sqrt{x^2+y^2}}, \tag{3.23}$$

$$s = \frac{x}{\sqrt{x^2+y^2}}. \tag{3.24}$$

(Continued on next page.)

Chapter 4

4.1 Householder reflectors

Example:

Find $H : Hx = ce_1$, $|c| = \|x\|_2$. Pick $H = I - 2uu^*$, $\|u\|_2 = 1$. Then: $H = H^*$ and $HH^* = I$, Hermitian unitary matrix

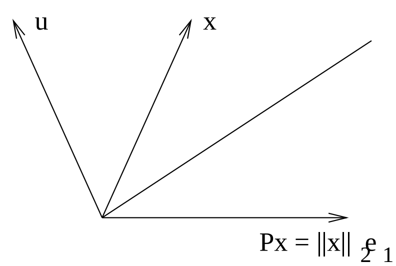

Figure 4.1: Householder Reflectors.

$u = ?$

$$H \cdot x = x - 2u(u^T x) = c \cdot e_1 \tag{4.1}$$

therefore, u is parallel to $x - ce_1$ (and is also of unit length by default).

$$u = \frac{x - Ce_1}{\|x\|_2} \tag{4.2}$$

which choice of C makes most sense?

$$C = -\text{sign}(x_1)e_1 \|x\|_2 \tag{4.3}$$

therefore, since $u = \frac{v}{\|v\|_2}$

$$v = x + \operatorname{sign}(x_1) e_1 \|x\|_2 \tag{4.4}$$

Applying a Householder: $(I - 2uu^T)A$. Naive implementation costs $2n^3$. Instead: $A - 2u(u^T A)$. Matrix-vector $(2n^2)$, outer product $(2n)$, and subtract $(2n^2) \Rightarrow 4n^2$.