

2.050J/12.006J/18.353J Nonlinear Dynamics I: Chaos, Fall 2012

Problem Set 8

Due at 12:01 pm on Friday, November 9th, in the box provided. No late psets are accepted. If you collaborated with other students in the class, list their names on the title sheet. The work that you submit must be your own.

Main concepts: Bifurcations of limit cycles, Poincaré map

Reading: Strogatz 8.2-3, 8.7

Problem 1: Global bifurcations of cycles

Strogatz 8.4.1

Problem 2: Global bifurcations of cycles

Discuss the bifurcations of the system $\dot{r} = r(\mu + 5 - r^2 + 6r)$, $\dot{\theta} = 1 + r$. Plot all the possible topologically different phase portraits in (x, y) .

Note: If there are 3 of them for values of $\mu = 7, 8, 9$, for example, to save time and paper you can plot them in subplots:

```
mu = [7,8,9];  
figure(123);  
for i = 1:3,  
subplot(2,2,i); streamslice(x,y,xdot,ydot);  
axis equal; grid on; xlabel('x'); ylabel('y'); title(['mu = ', num2str(mu(i))]);  
end
```

Problem 3: Poincaré maps

Given a vector-field:

$$\begin{aligned}\dot{x} &= x - y - x\sqrt{x^2 + y^2}, \\ \dot{y} &= x + y - y\sqrt{x^2 + y^2},\end{aligned}$$

1. Plot the phase portrait in MATLAB. Do you see any periodic solutions?
2. Rewrite the system in polar coordinates. Is there a stable limit cycle?
3. You should obtain in (2) that the cycle makes one revolution in $\Delta\theta = 2\pi$. Given that, integrate the r -equation between some value r_1 for which $\theta = 0$ to a value r_2 for which $\theta = 2\pi$, then write r_2 as a function of r_1 . You just obtained the analytical form of the Poincaré map: $r_{n+1} = f(r_n)$.

4. Use MATLAB or any other computational software to compute the first 10 iterations of the Poincaré map (i.e. $x_1 = f(x_0)$, $x_2 = f(x_1)$, etc.) starting from the initial conditions $r = 0.2$, $\theta = 0$. Submit those numbers. Do they approach any particular value?
5. In MATLAB plot the function that you just obtained and on top of it plot $r_{n+1} = r_n$. Use the commands `grid on`; `axis equal`; Print this out and iterate this map by hand (cobwebs) from the initial conditions $r = 0.2$, $\theta = 0$. Is it the same as in the previous question?
6. Repeat the previous two items for a trajectory starting at $r = 0.5$, $\theta = 0$. Does the trajectory converge to the same number? You just found a stable fixed point of a 1-dimensional map.
7. Compute the multipliers (Lecture on Tue).

Problem 4: Poincaré maps – iterating Poincaré maps

Given two Poincaré maps below

1. Print their graphs out as in Problem 3.5 (with the `grid on`; and the plotted diagonal $r_n = r_{n+1}$). Iterate them by hand using graphical technique of cobwebs and find the approximate values of the stable and unstable fixed points (which correspond to stable and unstable limit cycles).
2. Then iterate them using the computer from the same initial conditions as you did by hand, and this will give you numerical values of the fixed points. Report those.
3. What is the difference in the way that the iterations approach a stable fixed point in (a) versus in (b)? What condition on the derivative of the map at the fixed point would give you one type of approach versus the other one?

$$(a) \quad r_{n+1} = r_n + 0.5 \sin(r_n), \quad 0 \leq r_n \leq 8,$$

$$(b) \quad r_{n+1} = r_n - \ln(r_n), \quad 0 < r_n \leq 3.$$

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