7. Spinning, tumbling and rolling drops

7.1 Rotating Drops

We want to find $z = h(r)$ (see right). Normal stress balance on S:

$$\Delta P + \underbrace{\frac{1}{2}\Delta\rho\Omega^2 r^2}_{centrifugal} = \underbrace{\sigma \boldsymbol{\nabla}\cdot\boldsymbol{n}}_{curvature}$$

Nondimensionalize:
$\Delta p' + 4B_0\left(\frac{r}{a}\right)^2 = \boldsymbol{\nabla}\cdot\boldsymbol{n}$,
where $\Delta p' = \frac{a\Delta p}{\sigma}$, $\Sigma = \frac{\Delta\rho\Omega^2 a^3}{8\sigma} = \frac{centrifugal}{curvature} = =$ Rotational Bond number = const. Define surface functional: $f(r,\theta) = z - h(r) \Rightarrow$ vanishes on the surface. Thus
$\mathbf{n} = \frac{\boldsymbol{\nabla} f}{|\boldsymbol{\nabla}|} = \frac{\hat{z} - h_r(r)\hat{r}}{(1+h_r^2(r))^{1/2}}$ and $\boldsymbol{\nabla}\cdot\boldsymbol{n} = \frac{-rh_r - r^2 h_{rr}}{r^2(1+h_r^2)^{3/2}}$

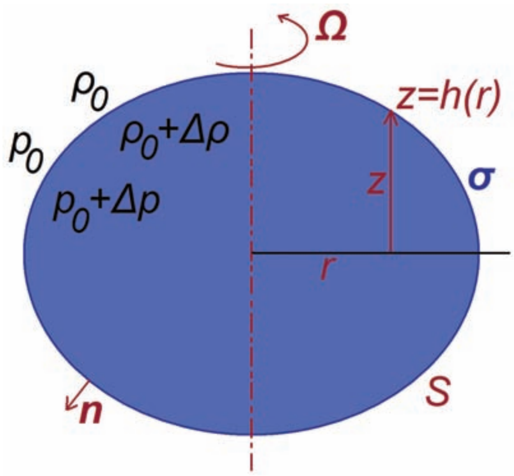

Figure 7.1: The radial profile of a rotating drop.

Brown + Scriven (1980) computed drop shapes and stability for $B_0 > 0$:

1. for $\Sigma < 0.09$, only axisymmetric solutions, oblate ellipsoids
2. for $0.09 < \Sigma < 0.31$, both axisymmetric and lobed solutions possible, stable
3. for $\Sigma > 0.31$ no stable solution, only lobed forms

Tektites: centimetric metallic ejecta formed from spinning cooling silica droplets generated by meteorite impact.

`Q1`: Why are they so much bigger than raindrops? From raindrop scaling, we expect $\ell_c \sim \sqrt{\frac{\sigma}{\Delta\rho g}}$ but both σ, $\Delta\rho$ higher by a factor of 10 $\Rightarrow$ large tektite size suggests they are not equilibrium forms, but froze into shape during flight.

`Q2`: Why are their shapes so different from those of raindrops? Owing to high ρ of tektites, the internal dynamics (esp. rotation) dominates the aerodynamics $\Rightarrow$ drop shape set by its rotation.

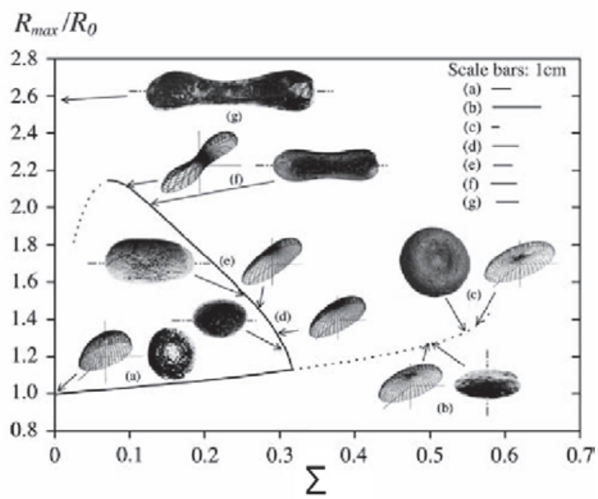

Figure 7.2: The ratio of the maximum radius to the unperturbed radius is indicated as a function of Σ. Stable shapes are denoted by the solid line, their metastable counterparts by dashed lines. Predicted 3-dimensional forms are compared to photographs of natural tektites. From *Elkins-Tanton, Ausillous, Bico, Quéré and Bush (2003).*

Light drops: For the case of $\Sigma < 0$, $\Delta\rho < 0$, a spinning drop is stabilized on axis by centrifugal pressures. For high $|\Sigma|$, it is well described by a cylinder with spherical caps. Drop energy:

$$E = \underbrace{\frac{1}{2}I\Omega^2}_{Rotational\ K.E.} + \underbrace{2\pi r L\gamma}_{Surface\ energy}$$

Neglecting the end caps, we write volume $V = \pi r^2 L$ and moment of inertia $I = \frac{\Delta m r^2}{2} = \Delta\rho\frac{\pi}{2}Lr^4$.

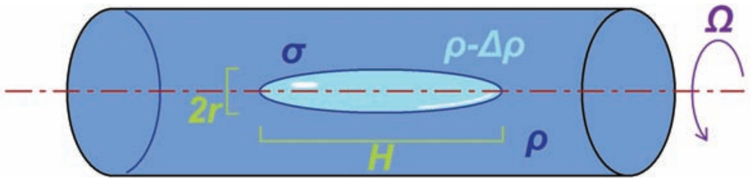

Figure 7.3: A bubble or a drop suspended in a denser fluid, spinning with angular speed Ω.

The energy per unit drop volume is thus $\frac{E}{V} = \frac{1}{4}\Delta\rho\Omega^2 r^2 + \frac{2\gamma}{r}$.
Minimizing with respect to r:
$\frac{d}{dr}\left(\frac{E}{V}\right) = \frac{1}{2}\Delta\rho\Omega^2 r - \frac{2\gamma}{r^2} = 0$, which occurs when $r = \left(\frac{4\gamma}{\Delta\rho\Omega^2}\right)^{1/3}$. Now $r = \left(\frac{V}{\pi L}\right)^{1/2} = \left(\frac{4\gamma}{\Delta\rho\Omega^2}\right)^{1/3} \Rightarrow$
Vonnegut's Formula: $\gamma = \frac{1}{4\pi^{3/2}}\Delta\rho\Omega^2\left(\frac{V}{L}\right)^{3/2}$ allows inference of γ from L, useful technique for small γ as it avoids difficulties associated with fluid-solid contact.
`Note`: r grows with σ and decreases with Ω.

7.2 Rolling drops

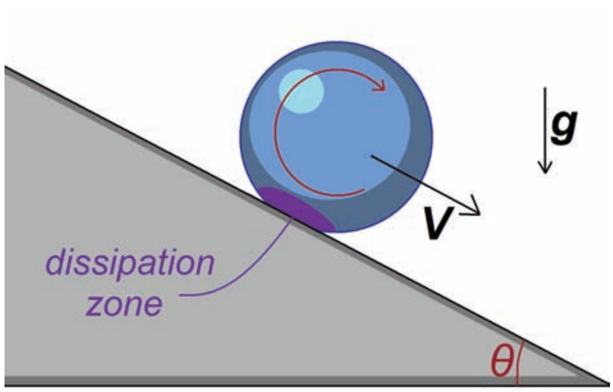

Figure 7.4: A liquid drop rolling down an inclined plane.

(*Aussillous and Quere 2003*) Energetics: for steady descent at speed V, $MgV\sin\theta$ =Rate of viscous dissipation= $2\mu\int_{V_d}(\boldsymbol{\nabla u})^2 \mathrm{d}V$, where V_d is the dissipation zone, so this sets $V \Rightarrow \Omega = V/R$ is the angular speed. Stability characteristics different: bioconcave oblate ellipsoids now stable.

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Fall 2010