Fourier Series Problem.

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This problem objective is to "experimentally" **study how Fourier Series converge.** For this purpose you should use the following MatLab scripts (which you can download from the WEB page, using the *MatLab for Fourier Series* link):

readmeFouSer.m   fourierSC.m   FSFun.m   FSoption.m   FSoptionP.m   heatSln.m

Put the scripts in a directory and start MatLab there. The help command will work as usual, e.g: help readmeFouSer gives a short description of all the scripts. Each script has its own detailed description. The *script you need is* fourierSC. The others (except for heatSln) are helper scripts.

Note: *you can alter* FSFun.m*, and write there any function for which you want to investigate the Fourier Series (this will allow you to go beyond the preselected options). Notice also that* fourierSC *makes tons of plots (they will show up one on top of the other, so you will need to "uncover" them).*

This is what you should do: Use the script fourierSC and report which "patterns" you see in the way Fourier Series converge. Experiment with the various choices. Look at the plots and think: what is happening? Many plots useful in figuring out how fast things converge (i.e. how fast do the Fourier coefficients vanish as $n \to \infty$) will be made by the scripts. Look at the plots, look for patterns and trends. Make an hypothesis as to what is happening and check it by further experimentation (use the script FSFun to produce functions where you can test your hypothesis). Write your conclusions in the answers. Describe the evidence for your conclusions — no proofs required, numerical evidence is enough. Think of it in the same way that you would think in the situation of a lab experimenter trying to figure out what happens in some problem.

You can use a few plots in your answer if you want, but do not go wild on this. Anything smaller than about 10^{-14} is numerical error, ignore it!

Graphs of the **power spectrum** will be made. These are plots of $\boxed{\sqrt{c_n^2 + s_n^2}}$ as a function of n, where c_n and s_n are the coefficients of $\cos(nx)$ and $\sin(nx)$ in the Fourier Series. They give information on "how important" the n-th mode is in the Fourier expansion. The name follows from the fact that in many physical situations one can interpret the square of the amplitude of the n-th Fourier coefficient as the amount of energy in the n-th mode of the solution (this is the case for the wave equation, for example).

THE END.

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