

Problem Set Number 6, 18.385j/2.036j

MIT (Fall 2014)

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October 22, 2014

Due Fri., October 31, 2014.

1 Problem 07.05.07 - Strogatz (Cell cycle)

Statement for problem 07.05.07

Tyson¹ proposed an elegant model of the cell division cycle, based on interactions between the proteins cdc2 and cyclin. He showed that the model's mathematical essence is contained in the following set of dimensionless equations

$$\frac{du}{dt} = b(v - u)(\alpha + u^2) - u \quad \text{and} \quad \frac{dv}{dt} = c - u, \quad (1.1)$$

where u is proportional to the concentration of the active form of a cdc2-cyclin complex, and v is proportional to the total cyclin concentration (monomers and dimers). The parameters $b \gg 1$ and $0 < \alpha \ll 1$ are fixed and satisfy $8\alpha b < 1$, while c is adjustable.

- A. Sketch the nullclines.
- B. Assume that $0 < \epsilon = \alpha b \ll 1$. Then show that the system exhibits relaxation oscillations when $c_1 < c < c_2$, for some constants c_1 and c_2 that you should determine approximately, using the fact that ϵ is small — exact expressions are also possible, and not too hard to get.
- C. Show that the system is excitable if c is slightly less than c_1 . Namely, the system has a globally attracting fixed point, but some (small, but not infinitesimal) disturbances can send the system on a long excursion through phase space before returning to the fixed point.

¹ J. J. Tyson, 1991, Modeling the cell division cycle: cdc2 and cyclin interactions, Proc. Nat. Ac. Sc. USA, **88**, 7328.

Remark 1.0.1 *The first quadrant is not invariant for this system, since $\dot{v} < 0$ for $u > c$. Thus solutions with $u > c$, and v too small, escape the first quadrant. Hence (1.1) is “un-biological” for $u > c$, and v too small.*

In your answer, concentrate only on the solutions for which $u \geq 0$ and $v \geq 0$ applies.

2 Justify the bead on a wire reduction

Statement: Justify the bead on a wire reduction

In Strogatz’ book, “bead on a wire” problems are often modeled by equations of the form

$$\dot{x} = f(x). \quad (2.1)$$

A more complete description of these systems typically has the form

$$m \frac{d^2 X}{dT^2} + b \frac{dX}{dT} = F f\left(\frac{1}{L} X\right), \quad (2.2)$$

where (m, b, F, L) are physical constants,² and f has no dimensions (T is time, with dimensions).

1. Introduce appropriate variables with no dimensions, and re-write (2.2) in terms of them.

Then use phase plane analysis to fully justify a condition under which (2.1) applies.

2. Estimate the size of the error in (2.1). That is, we can write $\dot{x} = f(x) + \mathcal{E}$, where \mathcal{E} is the error. Find a leading order estimation for \mathcal{E} .

3 Multiple scales and limit cycles #01

Statement: Multiple scales and limit cycles #01

Consider the equation

$$\frac{d^2 x}{dt^2} - \epsilon \cos x \frac{dx}{dt} + \frac{1}{\sqrt{\epsilon}} \sin(\sqrt{\epsilon} x) = 0, \quad \text{where } 0 < \epsilon \ll 1. \quad (3.1)$$

Use a multiple scales analysis to calculate the frequency, stability and amplitude of any limit cycle (the frequency up to the first correction beyond linear and the amplitude up to leading order).

THE END.

² For example: mass, damping coefficient, typical applied force, and typical length scale. But they can also be related to the capacitance, resistance, inductance and applied potential in a circuit. Or some other physics.

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Fall 2014

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