Assignment 4: Approximation Algorithms

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1. In the *Bin Packing* problem, we are given a set $S = \{s_1, \cdots, s_n\}$ where each $s_i \in [0, 1]$ and asked to "pack" elements of S into the minimum possible number of unit size bins. Design a 2-approximation algorithm for this problem, i.e. an algorithm which uses at most twice as many bins as the minimum possible.

2. Given an undirected graph $G = (V, E)$ a subgraph $G' = (V, E')$ such that $E' \subseteq E$ is called a 2-edge connected spanning subgraph if it has the property that there are at least two edge-disjoint paths in G' between any pair of vertices.

   (a) Show that the problem of finding a minimum 2-edge connected spanning subgraph is NP-hard.

   (b) Give an efficient 2-approximation for the problem, i.e. an algorithm that is guaranteed to find a solution with at most twice as many edges as the minimum.

3. Given a directed graph, an acyclic subgraph is a collection of edges $E' \subseteq E$ that contain no directed cycles.

   (a) Give a polynomial-time algorithm that finds an acyclic subgraph with at least half as many edges as the maximum.

   (b) Give an integer program, with a variable for each edge, to find the maximum cardinality acyclic subgraph in G.

   (c) Relax the integrality constraints to obtain an LP. How large can the integrality gap of the LP be? (In this case the integrality gap is defined as the ratio of the LP optimum to the the integer optimum.)

   (d) Give a polynomial-time separation oracle to solve the LP.