About PS3, problem 4, it should be pretty easy since you only need to plug $b = 1.5$ into Proposition 12.3.4 of RAP (*Real Analysis and Probability*, by R. M. Dudley, 2d edition, Cambridge University Press, 2002) and the series should converge fairly fast.

About applying Bretagnolle-Massart's Theorem 1.1 to the two-sample empirical process (as in PS3, problem 5):

First, here's some motivation. In the one-sample case there is the Dvoretzky-Kiefer-Wolfowitz-Massart inequality that tells us $\Pr(\sqrt{n}|(F_n - F)(x)| \geq u) \leq 2\exp(-2u^2)$ for any $u > 0$ and empirical distribution functions F_n for a distribution function F. So, this bound for the asymptotic distribution also applies for any finite n. In the two-sample case, though, it seems that no such inequality is known, so one needs to somehow "take apart" the two-sample quantity $\zeta_{m,n}$ to apply one-sample facts.

After following the problem statement and hints as far as they go, let's see how to bound $\zeta_{m,n} = \sqrt{\frac{mn}{m+n}}(F_m - G_n)$ as given in Problem 4. Under P_0, $F = G$ so we can write $F_m - G_n = (F_m - F) - (G_n - G)$. Let $\alpha_m(x) = \sqrt{m}(F_m(x) - F(x))$, an empirical process for F as treated by Bretagnolle and Massart in the uniform case. Let $\gamma_n(x) = \sqrt{n}(G_n - G)(x)$, an independent empirical process for $G = F$. Then approximate α_m by a Brownian process β_m which has the distribution of $y_{F(x)}$ for a Brownian bridge y_t, using Bretagnolle and Massart, and likewise approximate γ_n by a process B_n which also has the distribution of $y_{F(x)}$ and is independent of α_m and β_m. We then get

$$\zeta_{m,n} = \sqrt{\frac{n}{m+n}}[\beta_m + (\alpha_m - \beta_m)] - \sqrt{\frac{m}{m+n}}[B_n + (\gamma_n - B_n)].$$

Let $B_{m,n} = \sqrt{n/(m+n)}\beta_m - \sqrt{m/(m+n)}B_n$. Then, being a linear combination of two i.i.d. Gaussian processes with mean 0, where the sum of squares of the two coefficients is 1, $B_{m,n}$ also has the distribution of $y_{F(x)}$ for a Brownian bridge y_t. (This was used in the last lecture in proving that the asymptotic distribution for the two-sample Kolmogorov-Smirnov statistic under the null hypothesis is the same as for the one-sample statistic.) Since $n/(m+n) \leq 1$ and $m/(m+n) \leq 1$ we then get

$$|\zeta_{m,n}| \;\leq\; |B_{m,n}| + |\alpha_m - \beta_m| + |\gamma_n - B_n|.$$

Following the hints already given, we'd like to show that this is less than or equal to $\eta + (2-\eta)/2 + (2-\eta)/2 = 2$ except with probability $\leq 0.04 + 0.005 + 0.005 = 0.05$, where the bounds and probabilities correspond to the three respective terms.

Find η from RAP, Proposition 12.3.4, or the simpler bound using the leading term, for any $\eta > 0$, $\Pr(\sup_t |y_t| \geq \eta) \leq 2\exp(-2\eta^2)$. If you found in Problem 4 for $\eta = 1.5$ a probability ≤ 0.4 then 1.5 or a smaller value you could find by calculation will work. If you found a larger probability in Problem 4 you will need to take $\eta > 1.5$ but you need $\eta < 2$ for the method to work. It will be OK to find an η that works such that 10η is an integer (η has one digit after the decimal point), you don't need to find more decimal places.