Practice problems for Midterm 1

Note: this is not a representative exam (it has way more problems). The midterm will probably have about 7 or 8 problems. See the guidelines for further info.

1. Find the gcd of 621 and 483.
2. Find a solution of $621m + 483n = k$, where k is the gcd of 621 and 483.
3. Calculate 3^{64} modulo 67 by repeated squaring.
4. Calculate 3^{64} modulo 67 using Fermat's little theorem.
5. Calculate $\phi(576)$.
6. Find all the solutions of $x^3 - x + 1 \equiv 0 \pmod{25}$.
7. Find all solutions of $x^3 - x + 1 \equiv 0 \pmod{35}$.
8. Find the smallest integer N such that $\phi(n) \geq 5$ for all $n \geq N$.
9. Find two positive integers m, n such that $\phi(mn) \neq \phi(m)\phi(n)$.
10. True or false: two positive integers m, n are coprime if and only if $\phi(mn) = \phi(m)\phi(n)$. Give a proof or counterexample.
11. Give the definition of a reduced residue system modulo n.
12. State and prove the Chinese remainder theorem.
13. Show that $(n-1)! \equiv 0 \pmod{n}$ for composite n. [Hint: Make sure that your proof works for the case $n = p^2$, where p is a prime].
14. Solve the system of congruences

$$\begin{aligned} x &\equiv 1 \pmod{3} \\ x &\equiv 2 \pmod{5} \\ x &\equiv 3 \pmod{7} \end{aligned}$$

15. Let n be a positive integer. Show the identity

$$\sum_{i=1}^{n} i \binom{n}{i} = n2^{n-1}.$$

[Hint: differentiate both sides of the Binomial theorem, or manipulate the binomial coefficients.]

16. Calculate the order of 3 modulo 301.

18.781 Theory of Numbers
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