ALGEBRAIC NUMBER THEORY

LECTURE 9 NOTES

1. Secton 4.4

Proof of theorem 1: the bound on α should be $\alpha \geq 2^{n-r_1}(1/2\pi)^{r_2}|d|^{1/2}$.

2. Section 4.6

Solving the Brahmagupta-Pell-Fermat equation:
Write the continued fraction expansion of $\sqrt{d}$ as $[a_0, \overline{a_1, \ldots, a_k, 2a_0}]$. Then the smallest solution to

$$x^2 - dy^2 = \pm 1$$

is (a, b) where $a/b = [a_0, a_1, \ldots, a_k]$.

When $d \equiv 2$ or $3 \bmod 4$, this gives a fundamental unit for $\mathbb{Q}(\sqrt{d})$. When $d \equiv 1 \bmod 4$, this gives either the fundamental unit or its cube, and one can easily check whether a cube root exists in $\mathbb{Q}(\sqrt{d})$.

For a treatment of continued fractions see the book by Niven and Zuckerman.

3. Section 4.7

Proof of proposition 1: If A is an integral domain and $K = Frac(A)$ has characteristic 0, then Samuel asserts that K is a finite dimensional vector space over $\mathbb{Q}$. Note that this uses that A is integral over $\mathbb{Z}$ (being a finitely generated module over $\mathbb{Z}$ which is also a ring), which then implies that $K = \mathbb{Q}A \cong \mathbb{Q} \otimes_{\mathbb{Z}} A$.